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Comment

R. M. Dudley

David Pollard proved perhaps the most useful central limit theorem for empirical processes indexed by families of functions (Pollard, 1982), somewhat extended and exposited in Dudley (1984, Theorem 11.3.1) and Pollard (1984, Chapter 6). He has also been a leading worker at the interface of empirical processes and statistics, as in Pollard (1979) and the paper under discussion, with its 6 valuable references to his own work. Readers of Pollard (1985, 1989a), for example, will not need specifically econometric prerequisites, and they will find ideas not necessarily to be found elsewhere as far as I know.

On the foundations of empirical processes I would mention, as a step beyond my 1984 course which Pollard kindly cites, the paper Dudley (1985), which incorporates the new definition of convergence in distribution for stochastic processes due to Jørgen Hoffmann-Jørgensen (unpublished). This definition avoids the need to define any σ-algebra on large (nonseparable) spaces of bounded functions. Thus the process ν_n on a family of functions can converge *in* law without having a law on function space. Hoffmann's convergence in law is strong enough to imply existence of realizations converging almost surely or better, almost uniformly (also in Dudley, 1985) and so seems to be the "right" definition.

Pollard makes a good point that hypotheses on smoothness of parametrized families of functions $f(\cdot, \theta)$ with respect to θ can be weakened via empirical process theory. A related but different viewpoint is that of von Mises nonlinear, differentiable functionals of the empirical measure, which are beginning to be studied from the empirical process viewpoint (Sheehy and Wellner, 1988; Dudley, 1989). Let a family $\tilde{F}$ of functionals be, say, manageable with envelope 1, so that the supremum of $|\nu_n f|$ over $\tilde{F}$ is bounded in probability as $n \to \infty$. The supremum norm over $\tilde{F}$ then provides a norm for which functionals may be differentiable (Dudley, 1989). The many possible choices for such norms should help free von Mises theory from its focus on the real line as sample space and supremum of absolute differences of distribution functions as the main norm. It should also then be possible to make more use of Fréchet differentiability rather than compact (Hadamard) differentiability.

Instead of least absolute deviations, one can take an M-estimate of location (in $\mathbb{R}^k$ for any k), setting $\rho(x) = (c + |x|^2)^{1/2}$, $c > 0$, and minimizing $P_n\rho(x - t)$ with respect to t, where ρ is smooth but for small c is close to $|x|$. To treat laws P with infinite mean one can, as in Huber (1981, page 44), minimize $P(\rho(x - t) - \rho(x))$ in t, where the integrand is bounded in x for each t. Since ρ is strictly convex, the minimization is equivalent to finding the unique solution of $P\psi(\cdot - t) = 0$ where ψ is the gradient of ρ. The components of ψ, for any t, all belong to a uniformly bounded class of functions that can be shown to be manageable in much the same way as in the paper under discussion, Examples 5.5 and 5.6, even for $c = 0$ where the functions are no longer smooth.

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