class include the UMVUE ($S_e^2/I(J-1)$) and the

$$\text{MLE}\left(\min\left(\frac{S_e^2}{I(J-1)}, \frac{S_e^2+S_a^2}{IJ}\right)\right).$$

KMZ use a version of Stein's method to show that the MLE dominates the UMVUE. The authors go on to show, again using Stein's method, that any equivariant estimator of σ_e^2 which is greater than the sum of squares of all observations divided by $IJ + 2$ (i.e., $[IJY^2 + S_a^2 + S_e^2]/(IJ + 2)$) with positive probability, is inadmissible. This last result implies that there is additional information about σ_e^2 in the overall mean Y even though the variance of Y is a multiple of $\sigma_e^2 + J\sigma_a^2$ and not of σ_e^2.

Estimation of σ_a^2, of course, involves the additional wrinkle that the

$$\text{UMVUE } E\left(\frac{1}{J}\left(\frac{S_a^2}{I-1} - \frac{S_a^2}{I(J-1)}\right)\right)$$

is negative with positive probability. KMZ use Stein's method to investigate dominance relations among several estimators and show that the overall mean can sometimes be used to construct improvements. Portnoy (1971) and others have constructed Bayes equivariant estimators with good sampling properties. Loh (1986) has studied the problem of estimating σ_a^2/σ_e^2 using similar methods.

In this setup, since there is only one degree of freedom for the grand mean, the likely improvement is small (once one has selected a good equivariant estimator). It is possible that larger gains could occur in higher way mixed models where several degrees of freedom are available for the mean vector.

Presumably extensions of Brown's and Zidek's methods can be applied in these models and improved confidence intervals can be constructed as well.

ADDITIONAL REFERENCES

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Rejoinder

Jon M. Maatta and George Casella

To begin, we thank all discussants for their kind remarks and stimulating comments. This project was started to enhance our understanding of the topic, but also helped to improve our knowledge and perspective. As mentioned by several discussants, the scope of our work was limited. This work was an intentional decision, because our relatively narrow focus presented a reasonable size task, and allowed us a fuller understanding of one part of this complicated subject. Many of the discussants had similar concerns, and we will structure our rejoinder to respond to the major topics mentioned.

PRACTICAL CONSIDERATIONS

Though somewhat surprising to us, much concern was expressed over the magnitude of possible improvement. A major point was that the possible improvements in variance estimation seem small when compared to those possible in the estimation of means. This is true, but we feel that the improvement here is still worthy of consideration.

Berger expresses concerns about this and, in his inimitable way, anticipates some of our rejoinder. While the magnitude of improvement is small (as demonstrated by other discussants), it does increase in the generalized linear model case, which we do not consider "less realistic," but useful in practice. Very interesting calculations are provided by both Hwang and Rukhin, showing the limiting amount of improvement possible, approximately 25% in practical cases. Rather than interpreting these findings in the pessimistic way of Professor Hwang, however, we find more hope for future improvements (although we certainly agree that greater improvement seems possible in the estimation of means).

Some of our optimism is supported, and Hwang's pessimism negated, by the comments of George and Strawderman. They suggest that we have not yet fully exploited the structure of the problem. The risk (or interval length) improvement in variance estimation obtains when the means are close to the point to which we are shrinking. George and Strawderman each point out ways to shrink toward subspaces, and, further, George suggests that we can shrink toward multiple subspaces. Such estimators may provide substantial practical gains, since the region of improvement will be expanded. Another interesting possibility