that depends only on U is $U/(n+1)$. Considering estimators of the form $U\varphi(V/U)$, and using techniques analogous to those of Stein (1964), Pal and Sinha (1989) showed that the choice

$$\varphi(V/U) = \min\left\{\frac{1}{n+1}, \frac{1}{n+2}\left(1 + \frac{V}{U}\right)\right\}$$

produces an estimator that dominates $U/(n+1)$. For the same loss function, and using techniques analogous to those of Brewster and Zidek (1974), we can find a smooth estimator of λ^{-1} (MacGibbon and Shorrock, 1989). Because of the scale invariance of the problem, the distribution of V/U is independent of λ and this appears to be the only place in the argument where invariance plays a role.

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Comment

Andrew L. Rukhin

Maatta and Casella start their interesting paper with an analogy between the estimation of a multivariate normal mean and that of a normal variance. Indeed, in both of these problems a surprising inadmissibility phenomenon of a traditional and intuitively reasonable estimator has been discovered. However, each of these problems has distinctive features, and I would like to start by discussing two of them and then to comment on the asymptotic variance estimator and the variational form of Bayes estimators.

1. THE PROBLEM OF ESTIMATING A MULTIVARIATE NORMAL MEAN IS EASIER IN A SENSE

Let X have multivariate normal distribution $N_k(\mu, \sigma^2 I)$ and let S^2 be a statistic which is independent of X and such that S^2/σ^2 has a chi-squared distribution with ν degrees of freedom. This setting arises in a classical linear model where X represents the least squares estimator, and S^2, the residual sum of squares.

If μ is to be estimated under, say, quadratic loss, then one can demonstrate the inadmissibility of X for $k \geq 3$ using Stein's by now popular technique of integrating by parts. Indeed, if $\delta(X, S)$ is a smooth estimator, then one can obtain an unbiased estimator $D_\delta(X, S)$ of the risk difference

$$\Delta(\mu, \sigma) = [E\|X - \mu\|^2 - E\|\delta(X, S) - \mu\|^2]\sigma^{-2},$$

i.e.,

$$ED_\delta(X, S) = \Delta(\mu, \sigma).$$

It is also possible to choose δ so that $D_\delta \geq 0$, and hence this estimator, δ, improves on X.

In the problem of variance estimation, one can derive unbiased estimates of the risk difference for quadratic loss for the best equivariant estimator $S^2/(\nu+2)$. However, there is no alternative estimator for which this estimate is nonnegative. Conditioning on $\|X\|/S$ or representing the noncentral t-distribution, that of this statistic, as a Poisson mixture of central t-distributions is crucial for the inadmissibility proof. Notice that to estimate the risk difference Strawderman (1974) had used the so-called Baranchik lemma, which implies the nonnegativity of the expected value of a product of one monotone and one which changes signs.

2. RELATIVE RISK REDUCTIONS OF VARIANCE ESTIMATORS ARE SMALLER THAN FOR MEAN ESTIMATORS

It is known that in our setup for the crude James–Stein estimator

$$\delta(X, S) = \left[1 - \frac{(k-2)S^2}{(\nu+2)\|X\|^2}\right]$$

Andrew L. Rukhin is Professor, Department of Mathematics and Statistics, University of Maryland at Baltimore County, Baltimore, Maryland 21228.