

of Young and Harris (1990). Once the unimportant variables have been eliminated by simplification, then biplots can be used to display not only the distribution of observations in the two-dimensional multivariate space (as in the Weihs and Schmidli figures), but also the distribution of retained variables. This provides a more informative plot which displays relationships within the variables and between variables and observations, as well as within observations. The paper, the OMEGA pipeline, and the richness of the results would be strengthened with the inclusion of biplots.

The biplot can be extended in a very interesting way for redundancy analysis, as was originally proposed by Young and Sarle (1981). The extension uses the first two redundancy variates as the dimensions for a two-dimensional plot of the "redundancy plane." This is the plane in the predictor space which shares the most variance with the criterion space. A biplot can be constructed in this plane in the ordinary way, using the scores of the observations on the two redundancy variates as coordinates of observation-points, and the coefficients of the predictor variables on the redundancy variates as coordinates of the end points of predictor-variable-vectors which extend from the origin of the space. This biplot can be extended to become a *triplot* by adding to the biplot the projection of the criterion variables into the redundancy plane. They should be displayed as vectors. The plot of the redundancy plane now contains three kinds of information: the observations are represented as points, while the two sets of variables are represented as vectors.

The algebra underlying the redundancy triplot is as follows. The redundancy model is expressed by the equation $Y = XL$, subject to suitable restrictions on

L . Since L is nonnegative definite, it is the case that $L = AB$, and we can re-express the model by the fundamental RDA equation $Y = XAB$. The rank two approximation to the criterion variables Y is given by the approximation $Y \approx XA_2B$, where the subscript 2 indicates we are using only the two sets of linear combinations that correspond to the largest two eigenvectors. The redundancy model can now be re-written as $Y \approx Z_2B$, where $Z_2 = XA_2$. The values in Z_2 , which are the scores on the first two redundancy variables, are displayed as points in the triplot, whereas the values in A_2 (the coefficients of the predictor variables) and B_2 (the coefficients of the criterion variables) specify the endpoints of vectors emanating from the origin of the biplot.

ADDITIONAL REFERENCES

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Rejoinder

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We would like to thank the discussants for initiating the debate on our conceptual framework of interactive data analysis. Our responses cover five areas: the actual implementation of the OMEGA pipeline concerning software and methods, the data analysis example, possible extensions of the tool box, and a desirable ideal strategy.

SOFTWARE IMPLEMENTATION

The implementation of the OMEGA pipeline has always been, and remains, restricted by the graphical power of the underlying software (ISP). We have never

attempted to program our own graphics system. Therefore, the concept of the OMEGA pipeline goes far beyond our implementation (as described in Appendix 2). We were not intending to describe one more software tool, as Gower seems to assume, but rather a working implementation of a concept. Nevertheless, even the capabilities of the implemented version cannot be demonstrated on paper (see also Section 4.2). In fact, no real attempt was made to illustrate dynamics or to describe details of the software, like variable selection or interactive elimination of observations. Instead, we tried to demonstrate the power of the concept by showing what actions lead to which results.