expect that a discussion of data analytic strategies is helped by the precision obtained by casting strategies in terms of computational frameworks.

We would like to thank the authors for a stimulating paper and hope that this is not the end but the beginning of a discussion.

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Comment

Frank Critchley

It is a pleasure to welcome this paper by Weihs and Schmidli with its emphasis on the practical benefits which derive from combining classical dimensionality reduction methods with recent advances in interactive, dynamic graphics in a single integrated computing environment. At the same time, however pressing the practical need, asking for "a fairly general *single* routine strategy" (Section 1.1) for multivariate exploratory analysis seems, to me at least, to be asking for the moon. A more realistic objective might be to establish a framework of methods through which the user is guided by an expert system. We elaborate a little on this possibility below.

With one exception, my comments are of two types: possible extensions and remarks on the example. The exception is a detail which we dispose of first. In the context of resampling and Procrustes transformation (Section 3.7), the authors suggest that "it may be worth looking for analytic expressions derived from data disturbances analogously to Sibson (1979)." At least for PCA-COV and PCA-COR, some relevant formulae are given in Sections 3.6.2 and 6.3 of Critchley (1985). Note that the covariance matrix used there has divisor n. Trivial modifications apply when the divisor is $(n - 1)$. The formulae given are essentially expansions in inverse powers of $(n - 1)$. In practice, these expansions are usually truncated to obtain approximations. In this case, greater accuracy can be achieved by renormalization of the eigenvalues to sum to the easily computed perturbed trace and of the eigenvectors to have unit length. Exact orthogonalization is also possible.

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POSSIBLE EXTENSIONS

The following remarks are partly taken from the unpublished conference paper by Critchley (1987) on graphical data analysis. They relate principally to the dimensionality reduction methods employed.

1. In that paper I suggested that healthy progress requires constructive interaction between five ingredients: (a) important practical problems, (b) sufficient computing power, (c) a sound mathematical/statistical basis, (d) a good framework of methods, and (e) international cooperation. The present paper is an excellent example of the first three ingredients, while hopefully its publication in this format in this journal will encourage the last of these!

2. It is within the fourth ingredient that there is perhaps the greatest scope for fruitful extensions. The authors offer in Table 1 a classification of multivariate techniques in terms of two "dimensions": the preinformation required and the aspects of the data that are optimally represented. This framework of methods can be fruitfully extended by adding new methods (as the authors remark in Section 6) and also, we note here, by adding new "dimensions" to the classification of methods.

3. The methods currently considered can be characterized as corresponding to one of several possibilities on each of a (nonexhaustive) number of additional