

CONCLUDING REMARK

The above observations only confirm the importance of the subject discussed in the paper by Professor Shafer. I feel that the paper should be read and

discussed by all the statisticians who really care about the future of statistics.

I am very grateful to the Executive Editor, Professor Carl N. Morris, for inviting me to join the discussion.

Comment

David Aldous

Though asserting no interest in the foundational side of probability (thereby inviting a Keynesian riposte about being “enslaved to a philosophy long discredited”!), I found Professor Shafer’s article interesting and thought-provoking. Professor Shafer uses the word “probability” in a wise sense; reflecting my own interests, I use it mean “probability and its applications, *excluding* statistics.” Readers may judge for themselves whether my comments are relevant to statistics proper. Many of Professor Shafer’s comments concern teaching issues, whereas mine mostly address research.

1. RESEARCH-LEVEL APPLIED PROBABILITY

It is curious that there is no phrase “[adjective] mathematics” which adequately conveys the idea

(*M*) research whose conclusion is the statement and proof of a theorem

as opposed to

(*A*) research whose goal is answering a science question, using mathematics as a means rather than an end.

(I use “science” very broadly to mean some academic discipline in which mathematics can be used.) Though making distinctions between theory and applications is unfashionable and politically incorrect, I do see a distinction between seeking to make money at blackjack or the stock market and proving optimal strategy theorems; between designing airplane wings and proving theorems about air flow; between building reliable systems of components and proving theorems about increasing failure rates; between understanding molecular evolution and proving theorems about measure-valued diffusions.

Although an applied mathematician or statistician might claim to be doing both (*A*) and (*M*)—posing an extra-mathematical question and then answering it by

proving a theorem—the proportion of research papers that actually do both is extremely small. Most papers in (for example) *Annals of Statistics*, *Journal of Applied Probability*, *IEEE Transactions on Information Theory*, *SIAM Journal of Control and Optimization*, and *Journal of the Association for Computing Machinery* and much that is usually called “applied mathematics,” are plainly (*M*) but not (*A*). Good applied mathematics is like the unicorn: something we can all recognize but seldom actually see.

The part of (*M*) that is not traditional “pure mathematics” needs a name: I call it “theory-motivated-by-applications” (TMA) mathematics. Of current research involving probability, much more is TMA than is either (*A*) or pure mathematics. The key problem with research-level applied probability is the lack of agreed standards for evaluating TMA research. While this is not a pressing issue for most of us, it is for Mike Steele (as Editor of the new *Annals of Applied Probability*) and his associate editors. It would be unreasonable and divisive to erect high threshold standards for “serious math” and for “serious science” and insist that research exceeds one threshold or the other. There is a spectrum: at one extreme is serious math theory (at the level of *Annals of Probability*) with a rather vague connection to an application; at the other extreme is a serious science question which is solved by (to an expert theoretician) rather routine mathematics. Linear interpolation between these extremes is fine and constitutes what I regard as worthwhile applied probability. What concerns me is that, once a dozen people write papers on “probability methods in subject S,” a continuing subdiscipline is likely to be established. At best, this subdiscipline will produce results of interest to both the mainstream nonmathematical scholars in subject S and to workers in broader areas of theoretical and applied probability. At worst, it becomes an inward-looking clique ignored by everyone else. Of course this worst case also happens within theoretical disciplines, but there it is easier to detect. A cynic might say that applied probabilists can get away with claiming to theoreticians that they are solving science questions, and claiming

David Aldous is Professor, Department of Statistics, University of California at Berkeley, Berkeley, California 94720.