simple as possible, we further assume that the X_i have a density, and we take $S_0 = 0$ to have a clear convention. The convex hull H_n of $S_0, S_1, \cdots, S_n$ is a natural object that turns out to be intriguing both for its probability theory and geometry. A priori one might expect results on H_n to be difficult and incomplete, but—at least as far as first moments go—the theory is surprisingly easy and precise.

The first contributions to the theory of H_n are due to Spitzer and Widom (1961). Their seminal observation was to show that an ancient result of Cauchy could be combined with a purely combinatorial result of Kac (1954) to obtain an exact formula for the expectation of the length L_n of the boundary of H_n:

$$EL_n = 2 \sum_{k=1}^{n} E|S_k|/k.$$

From this beautiful formula one can obtain considerable information about EL_n, and, in particular, one can use it to show that $EL_n \sim c\sqrt{n}$ provided $EX_i^2 < \infty$ and $EX_i = 0$. This observation does not yet put us in the territory of the Poisson law—that comes later—but it does give the first suggestion of a counting law to be discovered.

The second paper to treat the geometry of H_n is due to Baxter (1961). This work shows, among other results, that the number N_n of sides of H_n has a remarkably simple expectation. In fact, it is just twice the nth harmonic number, i.e.,

$$EN_n = 2 \sum_{k=1}^{n} \frac{1}{k}.$$

In Snyder and Steele (1990), a common generalization of these results is given. If we let e_i, $i = 1, 2, \cdots$ denote the lengths of the faces of H_n and if f is any function, then provided both sides make sense we have the identity

$$E \sum_i f(e_i) = 2 \sum_{k=1}^{n} \frac{1}{k} Ef(|S_k|).$$

Naturally, this identity yields that of Spitzer and Widom by taking $f(x) = x$, and we can also get Baxter's identity just by taking $f(x) = 1$.

Now, here is where it may pay to start looking for a Poisson law. If we let f_n denote the indicator of an interval $[a_n, b_n]$, then for any given distribution of the X_i it is not hard to determine a_n and b_n so that for each n the sum $G_n = \sum_i f(e_i)$ satisfies $EG_n = \lambda > 0$. It may be most natural to take $a_n = 0$ in order to focus on the small faces of H_n. The variable G_n is nothing more than a sum of a random number of dependent random variables, the $f_n(e_i)$. Further, these variables do not seem all *that* dependent. Thus, there is a serious possibility of a Poisson approximation to the distribution of G_n.

Still, in this problem the Poisson law seems a long way away. The first moments were obtained through somewhat slippery trickery, and second moments do not seem to be open to more of the same. The Poisson law is honestly in play, yet the Chen–Stein method has far to come to meet the challenge. Can sufficient information be found on the second moments of G_n to complete the Chen–Stein program?

Comment

A. D. Barbour

The Chen–Stein method has added a new dimension to the techniques available for justifying Poisson approximations. In fields such a random graph theory (Bollobás, 1985, Chapter 4), extreme value theory (Smith, 1988; Holst and Janson, 1990) and spatial statistics (Barbour and Eagleson, 1984), where Poisson approximation plays an important role, the Chen–Stein method has already proved to be the best general approach, and its potential has by no means been exhausted. Its strengths are that it makes many sorts of weak dependence easy to handle, it gives explicit estimates of the accuracy of approximation, and it continues to give good results even when the expectation λ is large. The preceding survey illustrates the first two of these aspects admirably, but it gives rather less weight to the third, to which the following comments are addressed. For details and much more about the Chen–Stein method, see the forthcoming book of Barbour, Holst and Janson (1991).

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A remarkable feature of the Chen–Stein method is the form of the estimate of Theorem 1. When applied in the simplest setting, that of Theorem 0, it gives an error estimate no greater than $2\min(1, \lambda^{-1}) \sum_{i=1}^{n} p_{i,n}^2$. Were only an estimate of the form $c \sum_{i=1}^{n} p_{i,n}^2$ required, for some real c, it could be obtained