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## Comment

Theo Gasser, Christine Jennen-Steinmetz and Joachim Engel

Nonparametric curve estimation is coming of age, and it is thus timely to study the merits of various approaches. Two weighing schemes have been proposed in the kernel estimation literature, called "evaluation weights" and "convolution weights" by Chu and Marron. The goal of their paper is to give a balanced discussion of their merits, based on two complementary philosophies P1 and P2. We feel that the paper falls short of presenting a balanced discussion and often disregards philosophy P1, that is, looking for structure in a set of numbers. For many years the evaluation weights (due to Nadaraya and Watson) have been studied primarily for random design, the convolution weights for fixed design. Random design is defined and

treated adequately by the authors, while fixed design is represented by rather peculiar examples (see below). As is common (see, e.g., Silverman, 1984), we define a regular fixed design as  $x_i = F^{-1}((i - 0.5)/n)$ ,  $f = F'$ , where  $F$  is some distribution function with density  $f$ . Under standard assumptions, the asymptotic bias and variance for the two weighting schemes are as in Table 1, where  $M_2(K) = \int u^2 K(u) du$  and  $V(K) = \int K(u)^2 du$ .

### VARIANCE

The factor C in the variance of the convolution estimator is 1 for fixed and 1.5 for random design. Thus, we have an increase in variance for convolution weights with respect to the random design only; variances are asymptotically identical for regular fixed design. There is one fixed but not regular design of importance, that is, when we have multiple points, for example, due to rounding. It is easy to modify convolution weights for this design appropriately, and this has been done in our programs.

We are puzzled by the frequent use of the word *efficiency* in Section 3, when in fact only variance is

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