intercept of the linear model. This approach may be extended and the two model frameworks for equations (4.1) and (4.2) essentially integrated. Equation (4.5) may be generalized to allow all (or any) of the regression coefficients including the intercept to be random. Furthermore, small area level variables ($z_i$) may be used to explain some of the between small area variation:

$$y_i = x_i\beta_{1i} + e_i,$$
$$\beta_{1i} = z_i\gamma + \nu_i;$$

$X_i$ is the $N_i \times (p+1)$ matrix of unit level covariates (including an intercept) and $z_i$ is the $(p+1)\times q$ matrix of small area level variables. Here $\gamma$ is the vector of length $q$ of fixed coefficients and $\nu_i = (\nu_{i0}, \ldots, \nu_{ip})^T$ is a vector of length $p+1$ of random effects for the $i$th small area. In the general form the $\nu_i$ are independent between small areas but may have a joint distribution within each small area with $E(\nu_i) = 0$ and $V(\nu_i) = \Omega$:

$$\Omega = \begin{bmatrix} \sigma_0^2 & \sigma_{01} & \cdots & \sigma_{0p} \\ \sigma_{10} & \sigma_1^2 & \cdots & \sigma_{1p} \\ \vdots & & & \\ \sigma_{p0} & \sigma_{p1} & \cdots & \sigma_p^2 \end{bmatrix}$$

A special case is when the random effects are uncorrelated so that $\Omega$ is diagonal.

The use of area level variables, $Z_i$, to help explain the between area variation should help when the sample size in a small area is small. Also this more general model effectively integrates the use of unit level and area level covariates into a single model. Holt and Moura (1993) provide point estimates and expressions for MSE following the framework of Prasad and Rao (1990).

The use of extra random effects for the regression coefficients gives greater flexibility. If the unit level covariate is a set of dummy variables signifying group membership, for example, then this approach will allow a set of correlated and heteroscedastic random effects for the group means in each small area rather than a single random effect for all subjects.

The introduction of a random effect for the regression coefficient of a continuous covariate is likely to have more impact when the individual covariate values $x_{ij}$ are variable within each small area. Judging by the values displayed in Table 2 where the values of $x_{ij}$ vary greatly, it is possible that a more general model would provide even greater gains in precision for the empirical example which Ghosh and Rao consider.

# Comment

**Wesley L. Schaible and Robert J. Casady**

Professors Ghosh and Rao have provided us with an excellent, comprehensive review of indirect estimation methods which have been suggested for the production of estimates for small areas and other domains. They make a timely contribution by reviewing and comparing a number of new methods which have recently appeared in the literature as well as updating previous work on some of the more established approaches. Demographic methods, synthetic and related estimators, empirical Bayes estimators, hierarchical Bayes estimators and empirical best linear unbiased prediction methods are thoroughly discussed; evidence that the Bayes and empirical prediction methods have advantages over the others is presented. Special problems in the application of small area estimation methods are also addressed. This is an extremely important issue and additional discussion would have been desirable. In our comments, we will expand on this subject by discussing some of the characteristics of indirect estimators and some specific practical problems associated with their use. In addition, we will attempt to state in general terms what we believe to be the fundamental problem associated with the application of small area estimation methodology.

Very generally speaking, applications of indirect estimation methods fall into one of three categories:

1. An indirect estimator is used to estimate a population parameter;
2. an indirect procedure is used to modify a direct estimator of a population parameter (e.g., a direct estimator that incorporates indirectly estimated post-stratification controls or seasonal

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