out that the solution yields the constrained empirical Bayes estimates obtained by Cressie (1986), although no Bayes optimality criterion is invoked by the authors.

The multivariate version of (10) is

$$\hat{\theta} = A\tilde{\theta} + b, \tag{11}$$

where A is an $m \times m$ matrix and b is an $m \times 1$ vector. Upon specifying that $E(\hat{\theta}) = E(\theta)$ and $\mathrm{var}(\hat{\theta}) = \mathrm{var}(\theta)$, Cressie (1990b, 1992) obtains a multivariate constrained estimator. In the notation of (1), $\theta = \mu$, $E(\theta) = X\beta$, $\tilde{\theta} = y$, $E(y \mid \theta) = \theta$, $\mathrm{var}(y \mid \theta) = \Sigma$, and $\mathrm{var}(y) = \Sigma + \Gamma$. Then the multivariate constrained estimator for model (1), analogous to Spjøtvoll and Thomsen's, is given by (11), where

$$A = \Gamma^{1/2}(\Sigma + \Gamma)^{-1/2} \tag{12}$$

and

$$b = \{I - \Gamma^{1/2}(\Sigma + \Gamma)^{-1/2}\}X\beta. \tag{13}$$

Notice that $\hat{\theta}$ given by (11), (12) and (13) does not shrink y towards $X\beta$ as far as the Bayes estimator θ^* does (where $A = \Gamma(\Sigma + \Gamma)^{-1}$ and $b = (I - A)X\beta$).

In an elegant paper, Ghosh (1992) derives a multivariate constrained *Bayes* estimator for model (1):

$$\theta^{@} = \{a + (1 - a)\underset{\sim}{1}\underset{\sim}{1}'/m\}\theta^*, \tag{14}$$

where

$$a = \left[\mathrm{trace}\{(I - \underset{\sim}{1}\underset{\sim}{1}'/m)V\}\left(\sum_{i=1}^{m}(\theta_i^* - \bar{\theta}^*)^2\right)^{-1} + 1\right]^{1/2},$$

$$\theta^* = E(\theta \mid y) = \{\Gamma(\Sigma + \Gamma)^{-1}\}y + (I - \Gamma(\Sigma + \Gamma)^{-1}\}X\beta,$$

$$V = \mathrm{var}(\theta \mid y) = \Gamma\{I - \Gamma(\Sigma + \Gamma)^{-1}\}\Gamma.$$

The vector $\theta^{@}$ has the property that it minimizes $E(\sum_{i=1}^{m}(\theta_i - t_i)^2 \mid y)$ with respect to t and subject to conditions that match first and second sample moments of t with those same moments of θ conditional on y. Cressie's proposal given by (11), (12) and (13) does not invoke any optimality conditions and so is likely to be less efficient than Ghosh's estimator (14).

Constrained Bayes estimation for more general models, such as GLMs, is presented by Ghosh (1992), although from an essentially univariate point of view. Our earlier comment, that we do not have flexible ways to model lack of independence in nonlinear, nonnormal models, is equally appropriate here.

Finally, we agree with the authors' comment about the importance of small area estimation in medical geography. A good source for recent research in this area is the May 1993 Supplement Issue of the journal *Medical Care* (Proceedings of the Fourth Biennial Regenstrief Conference, "Methods for Comparing Patterns of Care," October 27–29, 1991). We are working on incorporating spatial variation and dependence into statistical methods for these and other small area estimation problems.

ACKNOWLEDGMENT

Partial support came from the Office of Naval Research Grant N00014-93-1-0001, NSF Grant DMS-92-04521 and the National Security Agency Grant MDA904-92-H-3021.

Comment

D. Holt

The paper by Ghosh and Rao is a valuable summary of recent developments using empirical Bayes and hierarchical Bayes methods for making small area estimates. The need for methods which make provision for local variation while pooling information across areas is well established. The review is a thorough appraisal of the methods and their properties, and the numerical results reinforce earlier results which demonstrate that these methods are preferable to others such as synthetic estimation and sample size dependent estimation.

The value of these approaches is not simply in their ability to provide point estimates for each small area which, on average, have better precision. A very important additional factor is that a measure of precision (MSE) and an estimator of this can be

D. Holt is Professor, Department of Social Statistics, University of Southampton, Southampton S09 5NH, United Kingdom.