

CORRECTION

BOOTSTRAP TESTS AND CONFIDENCE REGIONS FOR  
 FUNCTIONS OF A COVARIANCE MATRIX

BY RUDOLF BERAN AND MUNI S. SRIVASTAVA

*The Annals of Statistics* (1985) 13 95-115

Corollary 2 of Section 3.2 is incorrect because the remainder term in equation (3.15) does not behave as asserted. We wish to thank David Tyler for pointing out this error.

Suppose dimension  $p = 2$ , the cdf  $F$  has finite fourth moments, and is such that the eigenvalues of  $\Sigma_F$  both equal  $\nu(\Sigma_F) > 0$ . Let  $Z_F = \{z_{F,ij}\}$  be the random symmetric matrix defined in Section 2. For any constant symmetric  $2 \times 2$  matrix  $A = \{a_{ij}\}$ , let

$$K_F(A) = L[(y_F + w_F(A), y_F - w_F(A))'],$$

where

$$y_F = 2^{-1}(z_{F,11} + z_{F,22}),$$

$$w_F(A) = 2^{-1} \left\{ \left[ (z_{F,11} - z_{F,22} + a_{11} - a_{22})^2 + 4(z_{F,12} + a_{12})^2 \right]^{1/2} - \left[ (a_{11} - a_{22})^2 + 4a_{12}^2 \right]^{1/2} \right\}.$$

**COROLLARY 2.1.** *Under the model described above,  $L[J_{n,\lambda}(\hat{F}_n)]$  converges weakly to  $L[K_F(Z_F)]$  as  $n \rightarrow \infty$ . However, if  $\min(m, n) \rightarrow \infty$  while  $m/n \rightarrow 0$ , then  $J_{m,\lambda}(\hat{F}_n)$  converges weakly to  $L[\lambda(Z_F)]$ , in probability.*

Thus, in the equal roots case, the modified bootstrap estimate  $J_{m,\lambda}(\hat{F}_n)$ , which is based on bootstrap sample size  $m = o(n)$ , converges to the same limit law as does  $J_{n,\lambda}(F)$ ; but  $J_{n,\lambda}(\hat{F}_n)$  does not.

**PROOF.** Let  $\mu_F(r_1, r_2) = E_F(x_{11}^{r_1} x_{12}^{r_2})$ . Let  $D_4$  be the set of all nonnegative integer pairs  $(r_1, r_2)$  such that  $r_1 + r_2 = 4$ . Let  $\{F_n\}$  be any sequence of cdf's such that  $F_n \Rightarrow F$ ,  $\mu_{F_n}(r_1, r_2) \rightarrow \mu_F(r_1, r_2)$  for every  $(r_1, r_2) \in D_4$ , and  $n^{1/2}(\Sigma_{F_n} - \Sigma_F) \rightarrow A$ , a symmetric  $2 \times 2$  matrix. By a straightforward calculation using Theorem 1 and the formulae for the two eigenvalues,  $J_{n,\lambda}(F_n) \Rightarrow K_F(A)$ .

Let  $W_{n,F} = n^{1/2}(\hat{F}_n - F)$ ,  $Z_{n,F} = n^{1/2}(\Sigma_{\hat{F}_n} - \Sigma_F)$ , and  $t_{n,F} = \{(\mu_{\hat{F}_n}(r_1, r_2) - \mu_F(r_1, r_2)) : (r_1, r_2) \in D_4\}$ . The empirical processes  $\{(W_{n,F}, Z_{n,F}, t_{n,F})\}$  converge in law to a Gaussian process  $(W_F, Z_F, 0)$ . There exist versions  $\{(W_{n,F}^*, Z_{n,F}^*, t_{n,F}^*)\}$

Received August 1985.

