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DISCUSSION

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My comments center on three topics: the resampling algorithm of Section 7 as a bootstrap algorithm; criteria for assessing performance of a confidence set; and robustifying jackknife or bootstrap estimates for variance and bias. It will be apparent that I do not accept several of Wu's conclusions, particularly those concerning the bootstrap. The implied criticism does not diminish the paper's merit in advancing jackknife theory for the heteroscedastic linear model.

1. The bootstrap idea is a statistical realization of the simulation concept: one fits a plausible probability model to the data and acts thereafter as though the fitted model were true. Suppose that the errors $\{e_i\}$ in the linear model (2.1) are independent and that the c.d.f. of e_i is $F(\cdot/\sigma_i)$, where F has mean zero and variance one. Consistent estimates of the $\{\sigma_i\}$ and of F are not available, in general. Nevertheless, let $\hat{\sigma}_{n,i}$ be an estimate of σ_i , such as $\hat{\sigma}_{n,i} = |r_i|(1 - w_i)^{-1/2}$ or $\hat{\sigma}_{n,i} = |r_i|(1 - n^{-1}k)^{-1/2}$, and let \hat{F}_n be any c.d.f. with mean zero and variance one. The fitted model here is the heteroscedastic linear model parametrized by the estimates $\hat{\beta}_n$, $\{\hat{\sigma}_{n,i}\}$ and \hat{F}_n . The appropriate bootstrap algorithm, which I will call the heteroscedastic bootstrap, draws samples from this fitted model.

Section 7 of the paper describes just this resampling procedure, without recognizing it as a bootstrap algorithm suitable for the heteroscedastic linear model. The two bootstrap algorithms that are discussed critically in Section 2 are not even intended for the heteroscedastic linear model. The first is designed for the homoscedastic linear model; the second for linear predictors based on multivariate i.i.d. samples (Freedman (1981)).

Let $B_n(\beta, \{\sigma_i\}, F)$ and $V_n(\beta, \{\sigma_i\}, F)$ be the bias and variance of $g(\hat{\beta}_n)$ under the heteroscedastic model described in the preceding paragraphs. The ap-