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One can interpret one aspect of this very interesting paper as the testing of two null hypotheses. These may be called $H(\infty)$ ("independence" of rows and columns, called H_1 by the authors) and $H(1)$ (called H_0 by the authors) where $H(k)$ corresponds to a symmetric Dirichlet prior with parameter k. In the model adopted, for example, in Good (1983) and in my joint work with Crook, a mixture of all these symmetric Dirichlet prior was used, the mixture being regarded as a hyperprior density $\phi(k)$. (The idea dates back in part to Good, 1965, but was not developed much there.) Thus $H(k_0)$ can be regarded as the hypothesis for which $\phi(k)$ is the Dirac delta function $\delta(k - k_0)$. For multinomials, tests of $H(k)$ for $k = 1$, ½ (Jeffreys, 1946) and $1/t$ (Perks, 1947) were given in that paper, as well as a test for $U_k H(k)$ (the hypothesis that *some* symmetric prior is appropriate). Applications were made by Good (1983) to the problem of scientific induction, a point that could easily be overlooked because induction was not mentioned in the title of that paper.

The authors also interpolate a continuous infinity of hypotheses between $H(1)$ and $H(\infty)$, but in a manner different from that used by Crook and myself. Our interpolation was effected by using hypotheses $H(k)$ $(k > 0)$. For $k < 1$ these extrapolate beyond $H(1)$. It should be possible, but perhaps difficult, to determine which interpolation seems to give a better description of a given contingency table. Perhaps a very large sample size would usually be necessary to make this discrimination.

As a spin-off from the statistical work, the authors propose a new approximate formula for the famous old combinatorial problem of enumerating rectangular arrays with given margins. It might not be easy to decide under what circumstances their approximation is better than the ones proposed by Good (1976) and by Good and Crook (1977). My "ad hoc" improvement that the authors mention was an empirical adjustment to allow for the "roughness" of row or column totals. A really satisfactory formula should be algebraically symmetrical with respect to rows and columns thus avoiding giving two different answers. I wonder whether the authors have any further comments on this point.

The question of how much evidence regarding "independence" is contained in the marginal totals alone is of both logical and historical interest because Fisher's "exact test" for 2×2 tables ignored this evidence by conditioning on the marginal totals. The authors say, towards the end of their Section 3, that Crook and I "do not observe a very large difference between conditional and unconditional inferences (for this problem). This is because they use a variant of the uniform prior (2.11) which eliminates most of the supposed information in the margins." I think this comment requires some elaboration if it is not to be nearly tautological. We certainly selected a Bayesian model (the simplest usable one?) in which the row totals *alone*, or the column totals alone, convey no evidence against the null

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