

and hence may not be good pointers to multidimensional structure. Consideration of bivariate properties and, in particular, the search for two directions with maximum curvature of regression (Cox and Small, 1978, Section 4.2) may be more promising. Certainly that gives quite direct diagnosis of both smooth nonlinearity and groups of points away from a broadly linear form. There is much scope for empirical and theoretical study.

#### REFERENCE

COX, D. R. and SMALL, N. J. H. (1978). Testing multivariate normality. *Biometrika* **65** 263–272.

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This work makes a great contribution by introducing the unifying notion that projections are interesting if they minimize indices of randomness. Before, there was a sea of isolated, seemingly disjoint, ideas. Now there is some order, and a way of connecting the applied success stories of projection pursuit to more classical statistics. This often suggests new research projects.

One project involves notions of projection suitable for discrete data such as contingency tables and the analysis of preferences. I have introduced one such notion which involves projecting discrete data along “lines” of things like finite geometries. More formally, let  $X$  be a finite set (such as all binary  $k$ -tuples). Let  $f: X \rightarrow \mathbb{R}$  be a summary of the data (the proportion of students with a given pattern of correct/incorrect in a  $k$  item test). Let  $Y$  be a class of subsets of  $X$ . The Radon transform of  $f$  at  $y \in Y$  is the sum

$$\bar{f}(y) = \sum_{x \in y} f(x)$$

The class  $Y$  is a *projection base* if it partitions into  $y_1, \dots, y_t$  where each  $y_i$  is itself a partition of  $X$ .

In the example, the sets  $y_i^0 = \{x: x_i = 0\}$ , and  $y_i^1 = \{x: X_i = 1\}$  form a projection base. The Radon transform amounts to asking how many students answered the  $i$ th question correctly.

If the sets  $y_i$  are considered as lines in a geometry with points  $x$ , a projection base corresponds to the Euclidean axiom: for each point and line, there is a unique line through the point parallel to the given line. If  $X = \mathbb{R}^p$ , and  $Y$  is taken as all affine hyperplanes, the Radon transform gives ordinary projection.

A theory can be built in this generality. Many of the basic results seem to go through: for most data sets, most projections are close to uniform. Thus projections are interesting if they are far from uniform, and projection pursuit is forced on us.

I have analyzed several sets of discrete data using this approach. It leads to