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The authors are to be congratulated on this interesting and thought-provoking paper. They have raised a number of important questions and issues concerning additive model methodology. We will discuss some of these below. Throughout, our comments will be restricted to the case of symmetric smoothers having eigenvalues in $[0, 1]$.

**1. Exact and approximate concurvity.** This paper contains a thorough treatment of the fundamental issues of existence and uniqueness of solutions for the normal equations arising from additive model estimation. The authors show that these equations will have multiple solutions in certain cases. This raises questions as to how analyses should proceed in the presence of exact concurvity. Results from linear models would suggest that if $\mathbf{f}$ represents any solution to the normal equations, then one should only examine functionals $\mathbf{l}^t\mathbf{f}$ of the solution that are "*estimable*" in the sense that $\mathbf{l}^t\mathbf{g} = 0$ whenever $\hat{\mathbf{P}}\mathbf{g} = 0$. Such functionals are invariant under all choices of solutions to the normal equations and will have unique expectations. According to Theorem 5 of the paper, "estimable" functionals are provided by $np$-vectors in the orthogonal complement of the linear span of vectors $\mathbf{g}^t = (\mathbf{g}_1^t, \dots, \mathbf{g}_p^t)$ with $\mathbf{g}_j \in M_1(S_j)$ and $\mathbf{g}_+ = \mathbf{0}$. In particular we see that $\mathbf{f}_+$ is derived using "estimable" functionals.

Another approach to solving the normal equations for linear models of less than full rank is to reparameterize to obtain a full rank model. This is essentially what the authors have accomplished in Section 4.4 by extracting the projection parts from the smoothers, if linear dependencies are also eliminated from $M_1(S_1) + \cdots + M_1(S_p)$. The $\tilde{\mathbf{f}}_j$ are therefore obtained using "estimable" functionals and perhaps they are what should be studied when there is exact concurvity.

However, it seems to us that instances where an analysis should actually proceed in the presence of exact concurvity without some type of remedial action are rare. For example, in the case of smoothing splines, $M_1(S_j)$ is the linear span of the constant vector and $\mathbf{x}_j$. By Theorem 5 the concurvity space consists only of the constant vector unless the $\mathbf{x}_j$ are linearly dependent. In this latter case at least one of the variables should be dropped from the analysis to obtain meaningful estimates.

The real issue here seems to be approximate concurvity. As before we will draw an analogy with the linear regression case. In that setting approximate