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ASYMPTOTIC PROPERTIES OF MINIMIZATION ESTIMATORS FOR TIME SERIES PARAMETERS

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1. Introduction. For a strictly stationary stochastic process in discrete time, a second-order parameter associated with the process can be viewed as a function of the spectral distribution function. Various authors (e.g. Whittle, 1953; Walker, 1964; Ibragimov, 1967; Hosoya, 1974; Taniguchi, 1979, 1981; and Hosoya and Taniguchi, 1982) have considered a certain estimator defined as a minimization solution of an integral expression. This work is concerned with estimators which are solutions of integral minimizations of which the above are special cases (analogous to the relationship of MLE to M-estimation). Asymptotic properties are shown for these estimators with the establishment of probability one bounds being the most novel contribution. The approach taken in this paper is to show that such estimators have almost sure representations as integrals of kernel functions w.r.t. the sample spectral distribution function and to invoke known results for the latter-type estimators (Keenan, 1983). An application of the results to the construction of a whole family of strongly consistent estimators of the dimension of a parameter is given.

2. Background. Let $\{X_i, -\infty < i < \infty\}$ be a strictly stationary stochastic process with mean zero. We will assume throughout that

ASSUMPTION 1.

$$\sum_{v_1, v_2, \cdots, v_{k-1}=-\infty}^{\infty} |v_j| \, |c(v_1, v_2, \cdots, v_{k-1})| < \infty \tag{2.1}$$

for $j = 1, 2, \cdots, k-1$, $k = 2, 3, \cdots$, where $c(v_1, v_2, \cdots, v_{k-1})$ is the kth order cumulant of $\{X(0), X(v_1), X(v_2), \cdots, X(v_{k-1})\}$ (see Brillinger, 1975, Section 2.6). In the case of a Gaussian process this condition is satisfied if

$$\sum_{v=-\infty}^{\infty} |v| \, |c(v)| < \infty.$$

The absolutely continuous spectral distribution function of the X_n process will be denoted by $F(\lambda)$, $\lambda \in [0, 2\pi]$. We will assume throughout that $f(\lambda) = dF(\lambda)/d\lambda > 0$, $\lambda \in [0, 2\pi]$. For a sample $\{X_1, X_2, \cdots, X_n\}$, the sample spectral distribution is defined as

$$F_n(\lambda) = \frac{2\pi}{n} \sum_s I_n\left(\frac{2\pi s}{n}\right), \quad 0 < \frac{2\pi s}{n} \le \lambda \tag{2.2}$$

where $I_n(\lambda)$ is the sample periodogram

$$I_n(\lambda) = (1/2\pi n) \left| \sum_{t=1}^{n} X_t e^{-it\lambda} \right|^2, \quad \lambda \in [0, 2\pi]. \tag{2.3}$$

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