

## REPLY

We would like to thank the editor for inviting discussions on our paper focusing on the following issues: (1) Bayes or empirical Bayes approaches versus the classical decision theoretic approach; (2) the role of the loss function; (3) general adaptation of estimators and (4) practice. We would also like to thank the discussants for their constructive comments and suggestions for future research. Because of space limitations, we are able to respond only briefly to some of the issues raised by the discussants.

Like Professors Berger and Morris, we appreciate fully the advantage of using prior knowledge (if available) for the construction of Bayes and empirical Bayes estimators for the simultaneous estimation problem. In the Poisson means estimation problem under  $L_1$  loss, there are Bayes and empirical Bayes estimators dominating the usual ones (see Clevenson and Zidek, 1975, Tsui, 1978b, Ghosh and Parsian, 1981, and Ghosh, 1983) and such estimators belong to the general class of estimators proposed in the present paper. One drawback of Bayes procedures is that they very often suffer from lack of robustness against the choice of priors. Moreover, if one evaluates the risk performance of Bayes procedures (rather than the Bayes risk performance), one finds that such estimators can have risk considerably larger than the usual ones if the true parameter vector differs significantly from the prior mean. Our dominating estimators, however, cannot perform worse than the usual ones against *any* prior and are thus robust against misspecified priors (see Berger, 1982). Professor Morris has derived an interesting estimator (equation (10)) which does not depend on the explicit form of the prior, as long as the prior is exchangeable. However, the assumption that it is appropriate to use a linear Bayes rule must be justified, because the resulting estimator is *nonlinear*.

Professor Morris suggested the possibility of using loss function  $L_c$ , giving in equation (2) of his discussion, with general positive  $c_i$ 's. The loss function  $L_c$  was motivated by the problem of unequal sample sizes when estimating several Poisson means. Some improved estimators for these situations, where the  $c_i$ 's are general positive numbers, are provided by Tsui (1979b) with  $m_i = 0$  for all  $i$ , by Tsui and Press (1982), with  $m_i = m^* > 0$  for all  $i$ , and by Hwang (1982) for the more general case of  $m_i \geq 0$  for all  $i$ . Generalization of the results in the present paper to situations where  $L_c$  is appropriate follows directly from Theorems 3.1 and 4.1.

Both Professors Berger and Morris have noted the dependence of the dominating rules on the loss function. For estimating several Poisson means, Clevenson and Zidek (1975) obtained estimators dominating the usual ones under  $L_1$  and, more generally, under  $L_m$ , as long as  $m_i = m^* \geq 1$  for all  $i$ . At this point, it is not known whether there are estimators that uniformly improve upon  $\delta^0$  under both  $L_0$  and  $L_1$  when the underlying distributions are Poisson, or any other common distribution. The improved estimators' dependence on the loss function is found when distributions other than the Poisson are assumed (see, e.g., Berger, 1980). It should be noted that the empirical Bayes estimator (10) proposed by Professor Morris in general also depends on the choice of a loss function, even though it does not depend on the choice of  $c_i$  in his loss. Furthermore, the estimator (10) does not seem to us to be any simpler than our  $\delta^2$ , which has intuitive appeal (see Section 2).

On the question of how to choose the weights  $\theta_i^{-m_i}$  in the loss  $L_m$ , we merely point out that in many situations, inaccurate estimation for small  $\theta_i$ 's seems highly undesirable, and the choice of the  $m_i$ 's reflects our belief as to the relative severity of inaccurate estimation for different components.

Dr. Hudson advocates finding shrinkage estimators which have great reduction in risk in specific situations. We agree that this is important. However, a general theory is also of value because it provides a broad perspective on research in the entire area of

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