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HSU'S WORK IN MULTIVARIATE ANALYSIS

BY T. W. ANDERSON
Stanford University

From 1938 to 1945 Hsu published papers in the forefront of the development of the mathematical theory of multivariate analysis. It can be presumed that he was influenced by his proximity to R. A. Fisher, who was also at University College, London. After 1945 he lectured on multivariate analysis at Columbia University and the University of North Carolina, where he trained students who pursued research in this area. As a highly-trained mathematician Hsu promoted the use of matrix theory in statistical theory as well as proving new theorems about matrices.

A crucial element of multivariate theory is the distribution of the sample covariance matrix $\mathbf{S}$. When the p-component vectors are independently distributed each according to $N(\mathbf{0}, \mathbf{\Sigma})$, $(N-1)\mathbf{S} = \mathbf{A} = \Sigma_{\alpha=1}^{N}(\mathbf{X}_\alpha - \overline{\mathbf{X}})(\mathbf{X}_\alpha - \overline{\mathbf{X}})'$ has the so-called Wishart distribution with density (for $\mathbf{A}$ positive definite) of

$$w(\mathbf{A}|\mathbf{\Sigma}, n) = K(\mathbf{\Sigma}, n)|\mathbf{A}|^{\frac{1}{2}(n-p-1)}e^{-\frac{1}{2}\operatorname{tr}\mathbf{\Sigma}^{-1}\mathbf{A}}, \tag{1}$$

where $K(\mathbf{\Sigma}, n)$ is a constant depending on $\mathbf{\Sigma}$, its order p, and the "degrees of freedom" $n = N - 1$. For $p = 2$ the density of a_{11}, a_{22}, and $r_{12} = a_{12}a_{11}^{-\frac{1}{2}}a_{22}^{-\frac{1}{2}}$ was obtained by Fisher (1915) in his famous paper, which marked the beginning of rigorously derived exact small-sample distribution theory. Wishart's paper of 1928 derived the density (1) by use of a geometric argument, which was, roughly speaking, a generalization of that of Fisher. Since Wishart's publication many alternative proofs have been given. That of Hsu [5], based on algebra and analysis, is particularly elegant. To derive the density for p and n Hsu assumed it for $p - 1$ and $n - 1$. In addition to the matrix with this density there is needed a $(p-1)$-component normal vector and an n-component normal vector. With a little algebraic manipulation only the analytic derivation of the χ_n^2-distribution for the norm squared of the n-component vector is needed to complete the proof.

Mahalanobis, Bose and Roy (1937) approached the distribution of $\mathbf{A}$ by writing $\mathbf{A} = \mathbf{TT}'$, where $\mathbf{T}$ is lower triangular ($t_{ij} = 0$, $i < j$), and derived the distribution of the $p(p+1)/2$ elements of T, called "rectangular coordinates", from the distribution of $\mathbf{X}_1, \cdots, \mathbf{X}_N$. The (nonzero) elements of $\mathbf{T}$ were shown to be independent when $\mathbf{\Sigma} = \mathbf{I}$. Each off-diagonal element of $\mathbf{T}$ has the standard univariate normal distribution, and the ith (nonnegative) diagonal element has a χ-distribution with $n - i + 1$ degrees of freedom. Hsu's derivation of the distribution of rectangular coordinates in [8] is algebraic and analytic instead of the more geometric method of Mahalanobis, Bose and Roy. It is in the same spirit as the derivation of the Wishart distribution in [5].

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