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HSU'S WORK ON INFERENCE

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Hsu spent four years (1936-1940) at University College, London, where E. S. Pearson had recently succeeded his father in the chair of statistics and where, during the first two years, Neyman was a Reader in the Statistics Department. During this period Hsu wrote a remarkable series of papers on statistical inference which show the strong influence of the Neyman-Pearson point of view.

In 1938 Hsu's first two statistical papers appeared in Vol. II of the Neyman-Pearson edited *Statistical Research Memoirs*. The first of these [2] is concerned with what today is called the Behrens-Fisher problem. If X_i and $Y_j (i = 1, \cdots, m; j = 1, \ldots, n)$ denote samples from normal distributions $N(\xi, \sigma^2)$ and $N(\eta, \tau^2)$, Hsu considers the class of statistics $u = (\bar{Y} - \bar{X})^2/(A_1 S_x^2 + A_2 S_Y^2)$ where $S_X^2 = \Sigma(X_i - \bar{X})^2$ and $S_Y^2 = \Sigma(Y_j - \bar{Y})^2$. This reduces to u_1, Student's t, for $A_1 = A_2 = N/mn(N - 2)$ where $N = m + n$ and to the Behrens-Fisher statistic u_2 for $A_1 = 1/m(m - 1)$, $A_2 = 1/n(n - 1)$.

Hsu finds a series expansion for the density of u, and utilizes this to study the power function of the rejection regions $u \geqslant C$ in terms of the parameters $\theta = \tau^2/\sigma^2$ and $\lambda = (\eta - \xi)^2/(\frac{\sigma^2}{m} + \frac{\tau^2}{n})$. It is an exact (not asymptotic) analysis, described by Scheffé (1970) as "a model of mathematical rigor". In the process, he obtains stochastic bounds for u_2 which were later taken up independently and generalized by Hájek, Lawton and others (cf. Eaton and Olshen (1972)). Hsu's main conclusion, obtained by a combination of his analytical study with some numerical work, is that for $\lambda = 0$ and varying θ neither u_1 nor u_2 control the rejection probability at all well (except when $m = n$) although of the two, u_2 is less sensitive to variation of θ.

In the second paper [3], Hsu treats the question of optimal estimators of the variance σ^2 in the Gauss-Markov model. In the spirit of the Gauss-Markov theorem, he considers estimators Q which are (a) quadratic and (b) unbiased. In addition he imposes the restriction (c) that the variance of Q be independent of the unknown means. (This is a forerunner of the condition he imposed in [12] for the power function of analysis of variance tests).

Hsu then obtains a necessary and sufficient condition for the usual unbiased estimate S^2 of σ^2 to have uniformly minimum variance within this class of estimators. He illustrates the condition on a number of examples and, in particular, shows that S^2 has the desired property in the one-sample case. The problem was

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