

where

$$k(\tau) = \frac{1}{\pi} \int_0^{\infty} \frac{1}{(1+y^{2m})} \cos \tau y \, dy,$$

illustrating the “bandwidth” role of  $\lambda$ . (See [1].)

Moore and Yackel [5] have made a detailed comparison of window vs.  $k$ -NN type density estimates and conclude (not surprisingly) that one does better with  $k$ -NN estimates near  $x$  where  $h(x)$  is small (and presumably vice-versa). A direct comparison of practical  $k$ -NN type estimates vs. window type estimates for  $E(Y|X=x)$  must of course include the prescription for choosing  $k$  or  $\lambda$  as well as for choosing the shape, e.g., uniform, triangular or quadratic examples as given by Professor Stone, or as determined by  $Q$  here. Any  $Q$  within the same equivalence class (in the sense of [9]) will give the same (asymptotic) results, so within a class, computational ease can be the criteria. To choose from among a finite number of representatives of equivalence classes compute  $\min_{\lambda} V(\lambda)$  or  $\min_{\lambda} \hat{R}(\lambda)$  for each representative and take the minimizer over the representatives tried.

#### REFERENCES

- [1] COGBURN, R. and DAVIS, H. T. (1974). Periodic splines and spectra estimation. *Ann. Statist.* **2** 1108–1126.
- [2] CRAVEN, P. and WAHBA, G. (1976). Smoothing noisy data with spline functions: Estimating the correct degree of smoothing by the method of generalized cross-validation. Unpublished.
- [3] HUDSON, H. M. (1974). Empirical Bayes estimation. Technical Report 58, Dept. Statist., Stanford Univ.
- [4] KIMELDORF, GEORGE and WAHBA, GRACE (1971). Some results on Tchebycheffian spline functions. *J. Math. Anal. Appl.* **33** 82–95.
- [5] MOORE, D. S. and YACKEL, J. W. (1976). Large sample properties of nearest neighbor density function estimators. Mimeo series 455, Dept. Statist., Purdue Univ.
- [6] WAHBA, G. (1975). A canonical form for the problem of estimating smooth surfaces. Technical Report 420, Dept. Statist., Univ. of Wisconsin-Madison.
- [7] WAHBA, G. (1977). Practical approximate solutions to linear operator equations when the data are noisy. *SIAM J. Num. Anal.* **14**, No. 4. To appear.
- [8] WAHBA, G. (1976). A survey of some smoothing problems and the method of generalized cross-validation for solving them. Technical Report 457, Dept. Statist., Univ. of Wisconsin-Madison. *Proc. Symp. Appl. Statist.* (P. R. Krishnaiah, ed.). To appear.
- [9] WAHBA, G. (1974). Regression design for some equivalence classes of kernels. *Ann. Statist.* **2** 925–934.
- [10] MALLOWS, C. L. (1973). Some comments on  $C_p$ . *Technometrics* **15** 661–675.

#### REPLY TO DISCUSSION

First I wish to thank an Associate Editor handling the paper for suggesting that it be used for discussion. I also wish to express my gratitude to him and the other discussants for the wide variety of interesting, thought provoking and uniformly constructive comments and to the Editor, Richard Savage, for his help in improving the accuracy, style and readability of the paper.

Cover wonders why continuity requirements are not needed for consistency.