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# CORRECTION TO "ON THE EXISTENCE OF A MINIMAL SUFFICIENT SUBFIELD"

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We are grateful to Professor Harald Luschgy for pointing out an inaccuracy in the above paper [10]. Theorem 3.1 of [10] is an incorrect translation of Theorem 2.2 of Krickeberg (1960). Theorem 3.1 of [10] becomes correct under the additional assumption that each subfield $U_\tau$ contains all $\mu$-null sets (this follows from Theorem 6.1 of Hunt [11] and the null set assumption; see also the discussion by Luschgy [12]). The auxiliary Theorem 3.2 of [10], whose proof depended on the incorrect version of Theorem 3.1, is also incorrect; Example 3 of Burkholder (1961) gives a counterexample.

Fortunately the main result, Theorem 2.3 of [10], is correct and can be proved without appealing to Theorem 3.2. The second and third paragraphs of the proof of Theorem 2.3 in Section 4 of [10] should be replaced by the following three paragraphs. The notation used here is that used in [10].

Since each $U_\tau$ is sufficient for $M$, there exists a $U_\tau$-measurable function $h_\tau$ in $B$ such that $h_\tau$ is a version of $P[A \mid U_\tau]$ for each $P$ in $M$. For fixed $P$, the collection of subfields $\{U_\tau \vee N_p\}$ is directed downward by inclusion, and the collection $\{(h_\tau, U_\tau \vee N_P)\}$ is a $P$-uniformly integrable martingale relative to $P$: if $U_\tau \subseteqq U_\rho$ then

$$\begin{aligned} E_P[h_\rho \mid U_\tau \vee N_P] &= E_P\{P[A \mid U_\rho] \mid U_\tau \vee N_P\} \\ &= P[A \mid U_\tau \vee N_P] \qquad \text{a.e.} \quad [P] \\ &= h_\tau \qquad \text{a.e.} \quad [P] \end{aligned}$$

by Lemma 4.8 of Bahadur (1954). Since $U_\tau \vee N_P \supseteqq N_P$ for each $\tau$, the corrected statement of Theorem 3.1 of [10] implies that there exists a function $f_P$, measurable with respect to the subfield $\bigcap_\tau (U_\tau \vee N_P)$, such that $\lim_\tau ||h_\tau - f_P||_P = 0$, where $||\cdot||_P$ denotes the $L_1(P)$-norm. Truncating if necessary, we take each $f_P$ to satisfy $0 \leqq f_P \leqq 1$ on $X$, i.e., $f_P$ is in $B$.

We now will apply Lemma 1.2 of Pitcher (1965) to show that the element $(f_P)$ of $\prod B(P)$ is countably coherent. For any fixed $1 < r < \infty$ and $P$ in $M$, let $W_r(P)$ denote the weakly topologized unit ball in $L_r(P)$, let $\prod_{P \text{ in } M} W_r(P)$ denote the Cartesian product space endowed with the Tychonoff (product) topology, and let $B_r = \bigcap_{P \text{ in } M} W_r(P)$, and let $i_r : B_r \to \prod W_r(P)$ be defined by $i_r(h) = (h, h, \cdots)$ (see the bottom of page 598 of Pitcher (1965)). For $h$ in $L_s(P)$ (where $r^{-1} + s^{-1} = 1$) we have

$$\begin{aligned} |\textstyle\int h_\tau h \, dP - \int f_P h \, dP| &\leqq (\textstyle\int |h_\tau - f_P|^r \, dP)^{1/r} (\int |h|^s \, dP)^{1/s} \\ &\leqq ||h_\tau - f_P||_P^{1/r} (\textstyle\int |h|^s \, dP)^{1/s} \end{aligned}$$

since $|h_\tau - f_P| \leqq 1$. Therefore the net $\{i_r(h_\tau)\}$ converges to $(f_P)$ in the product