

## INEQUALITIES FOR SYMMETRIC SAMPLING PLANS I<sup>1</sup>

BY SAMUEL KARLIN

*The Weizmann Institute of Science  
and Stanford University*

**Introduction.** In recent years there has been much interest in evaluating certain probabilistic quantities arising in sampling with and/or without replacement, from finite populations, e.g., Korwar and Serfling (1970), Lanke (1972), Sen (1970), Serfling (1973), Kemperman (1973), and others. For these special sampling procedures a variety of inequalities have been derived for the moments of the sums and maximum of partial sums based on the observed sample.

Rosén (1972) investigated the validity of a central limit theorem for weighted sums of observation arising from quite general sampling schemes. In an earlier paper Rosén (1967) established a convexity type inequality comparing the expectations of certain functionals evaluated with respect to a "symmetric" sampling plan and for the special plan of sampling without replacement. It was this earlier paper that kindled our interest. The present work develops a variety of inequalities for functionals associated with sampling plans which can be interpreted as results on multivariate ordering relationships among certain distributions.

The organization of the contents is as follows. In Section 1 relevant definitions and preliminaries are introduced. Several important classes of sampling procedures are delimited including the "symmetric" sampling schemes, "random replacement policies," sampling procedures based on special partitionings of the sample space, sampling plans involving a prescribed number of distinct observations with given multiplicities, conditional sampling procedures and other

---

Received March 1973; revised January 1974.

<sup>1</sup> Research supported in part by ONR Contract N00014-67-A-0112-0015 at Stanford.