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CORRECTION TO

ADMISSIBLE ESTIMATORS, RECURRENT DIFFUSIONS, AND INSOLUBLE BOUNDARY VALUE PROBLEMS

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In the above paper (*Ann. Math. Statist.* **42** 855–903) we introduced the diffusion $\{Z_t\}$ defined on E^m as having local mean $\nabla \log f^*$ and local variance 2I.

C. Srinivasan (private communication) has pointed out the following difficulty with this definition and our usage of $\{Z_t\}$. Since $\nabla \log f^*$ is a C^∞ function Z_t may always be defined locally (see e.g. McKean (1969)). However it may happen that $\{Z_t\}$ "explodes" in a finite time. To be precise, define the random time (time of explosion) as

$$\mathscr{T}^x = \sup_{R\to\infty} \inf\{t : Z_t^x \geqq R\}\,.$$

Either $\mathscr{T}^x = \infty$ w.p. 1, or not. In the latter case our definition of $\{Z_t\}$ is defective for $t \geqq \mathscr{T}$. To repair the definition let $Z_t = \infty$ if $t \geqq \mathscr{T}$. Z_t is then a well-defined diffusion on $E^m \cup \{\infty\}$.

Let us make some remarks to clarify the effect of this new definition of $\{Z_t\}$.

(1) *All of the main results of the paper remain true with this new definition of* $\{Z_t\}$ *exactly as they are stated in Brown* (1971); *except for Theorem* 4.3.1 *which requires a minor change*. (See (5, vi) below.) This includes all the results labeled as Theorems or Corollaries. However, some of the Lemmas must be modified. See below.

(2) If $\Pr\{\mathscr{T}_E^x < \infty\} > 0$ for some $x \in E^m$ then $\Pr\{\mathscr{T}^x < \infty\} > 0$ for all $x \in E^m$. In this case $\{Z_t\}$ is transient according to the definition (4.1.4) (which remains appropriate even with the above, revised definition of $\{Z_t\}$). These facts can be deduced from the discussion in McKean (1969, Section 4.4) and from previously described properties of $\{Z_t\}$.

(3) The situation $\Pr\{\mathscr{T}^x < \infty\} > 0$ is possible for diffusions of the type considered here; but only if $\sup\{f^*(x) : |x| = r\}$ increases exceedingly rapidly as $r \to \infty$. As an example suppose $m = 1$ and $f^*(\{k\}) = e^{k^2/2}/k!$ $k = 0, 1, 2, \cdots$. Then $f^*(x) = \exp(e^x - x^2/2)$. Hence $\nabla \log f^*(x) = e^x - x$. It follows from Feller's test for explosion that $\Pr\{\mathscr{T}^x < \infty\} = 1$. (See e.g. McKean (1969, page 65).)

On the other hand, if $\limsup_{r\to\infty} \{\|\nabla \log f^*(x)\| : \|x\| = r\}/r < \infty$ then $\Pr\{\mathscr{T}^x < \infty\} = 0$, by Hasminskii's test (McKean (1969, page 102)). It can be shown that this is the case if $\int_{|x|<r} dF(x) = O(e^{kr})$ as $r \to \infty$ for some $k < \infty$.

(4) The only formal result in our paper which directly uses $\Pr\{\mathscr{T} < \infty\} = 0$ in its statement or proof is Lemma 4.2.1. Some later results use this Lemma in their proof but otherwise make no use of $\Pr\{\mathscr{T} < \infty\} = 0$. Lemma 4.2.1 remains correct if $m = 1$, or if $K = E^m$. This is easy to check. Therefore: