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It is a pleasure to add my congratulations to Luke Tierney on his important paper, which not only provides a sound theoretical basis for the use of Markov chain Monte Carlo (MCMC) methods in Bayesian inference but also gives valuable practical guidance. It is noteworthy that versions of the paper have been available for a couple of years now and have already proved to be highly influential. Subsequent developments, often involving the author himself, have been extremely rapid and I hope he will take the opportunity to tell us something about these in his rejoinder. For example, regeneration methods, which are only briefly discussed in the paper, have been the subject of considerable progress [e.g., Mykland, Tierney and Yu (1995)]. In the very recent work of Geyer and Thompson (1993), they are used cleverly on a succession of chains, ranging from "hot" (e.g., independence) to "cold" (the distribution of interest). The idea is that swaps into the hot chain, which can be sampled exactly and hence forgetfully, provide the regeneration points. These authors also show how to adapt their strategy to a single chain by subsampling from a randomly varying distribution between regenerations, so that no form of burn-in is required.

Markov random fields and Gibbs. I particularly welcome Tierney's survey of a wide variety of different MCMC algorithms, including hybrid implementations to which I shall return later. It is easy to be seduced into using the Gibbs sampler as one's only Bayesian inference machine, as I know only too well in spatial applications [Besag (1989), Besag and Mollié (1989), Besag and York (1989) and Besag, York and Mollié (1991)]. In fact, Gibbs has extra allure in spatial statistics. The reason is that a standard means of obtaining a distribution π for a random vector $X = (X_1, \dots, X_n)$, where each X_i is associated with a fixed spatial location (or *site*) i, is in terms of a Markov random field formulation [Besag (1974)]. This requires that one examines each site in turn and specifies the "full" conditional distribution $\pi(x_i \mid x_{-i})$ there; these conditionals are called *local characteristics* in spatial statistics. Such a conditional probability approach to spatial interaction was advocated by Bartlett (1967), as part of his presidential address to the Royal Statistical Society. There are two immediate questions. Do the local characteristics determine π and what

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