

INVITED PAPER

COMPOUND DECISION THEORY AND EMPIRICAL BAYES METHODS

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1. Introduction. Compound decision theory and empirical Bayes methodology, acclaimed as “two breakthroughs” by Neyman (1962), are the most important contributions of Herbert Robbins to statistics. The purpose of this paper is to provide a brief description of his work in these two intimately connected fields, its impact and a number of important related developments.

Robbins introduced compound decision theory in 1950 at the Second Berkeley Symposium on Mathematical Statistics and Probability. Compound decision theory concerns a sequence of independent statistical decision problems of the same form. Its basic thrust is the possibility of gaining substantial reduction of total risk by allowing statistical procedures for the individual component problems to depend on the observations in the entire sequence. It demonstrates, against naive intuition, that stochastically independent experiments are not necessarily “noninformative” to each other in statistical decision making.

Five years later, at the Third Berkeley Symposium, Robbins developed empirical Bayes (EB) theory. EB concerns experiments in which the unknown parameters are i.i.d. random variables with an unknown common prior distribution. EB methodologies provide statistical procedures which approximate the ideal Bayes rule for the true model, so that the goal of the Bayesian inference is nearly achieved without specifying a prior. EB procedures usually perform well conditionally on the unknown parameters and thus provide solutions to compound decision problems. EB methods also find applications in problems with more complex structures and for inference about multivariate and infinite-dimensional parameters in a single experiment.

Compound decision theory and EB have had great influence on modern statistical thinking and practice. Since Robbins’ pioneering papers, EB methods have been applied in a wide range of paradigms and to numerous real-life problems; cf. Neyman (1962), Cover (1968), Copas (1972), Carter and Rolph (1974), Simar (1976), Efron and Morris (1977), Van Ryzin and Susarla (1977), Susarla and Van Ryzin (1978), Rubin (1980), Hoadley (1981), Morris (1983),

Received December 2001; revised June 2002.

AMS 2000 subject classifications. Primary 62C12, 62C25; secondary 62G08, 62G05.

Key words and phrases. Empirical Bayes, compound decision theory, nonparametric inference, prediction.