

NOTE

CORRECTION TO

“A COUNTEREXAMPLE TO PEREZ’S GENERALIZATION OF THE SHANNON-McMILLAN THEOREM”

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In [1] we presented an example which was stated to be a counterexample to Theorem 2.3 of [2]. This is not true since an assumption implicit from page 553 of [2] was overlooked. In this correction note, by modifying the technique of [1], we obtain a correct counterexample. We remark that Dr. Perez has recently repaired his Theorem 2.3 by adding an additional assumption; see [3].

Let (Ω, \mathcal{F}) be the measurable space where Ω is the set of doubly infinite sequences of integers and \mathcal{F} is the usual product sigma-field. For each integer i , let X_i be the projection from Ω onto the i th coordinate. If i, j are integers such that $i \leq j$, let $\mathcal{F}_{i,j}$ be the sub-sigmafield of \mathcal{F} generated by X_i, X_{i+1}, \dots, X_j . If P is a probability measure on \mathcal{F} let P_n be the restriction of P to $\mathcal{F}_{1,n}$, $n = 1, 2, \dots$.

To provide a counterexample to Theorem 2.3 of [2] we construct probability measures P, Q on \mathcal{F} such that

- (a) P, Q are stationary;
- (b) P_n is absolutely continuous with respect to Q_n , $n = 1, 2, \dots$;
- (c) $\lim_{n \rightarrow \infty} \int_{\Omega} n^{-1} \log (dP_n/dQ_n) dP$ exists and is finite;
- (d) for each $E \in \mathcal{F}_{0,0}$, $\lim_{n \rightarrow \infty} Q(E | \mathcal{F}_{-n,-1})$ exists a.e. [P], and there exists a probability measure P' on $\bigvee_{i \leq 0} \mathcal{F}_{i,0}$ such that

$$P'(F \cap E) = \int_F \lim_{n \rightarrow \infty} Q(E | \mathcal{F}_{-n,-1}) dP,$$

for each $F \in \bigvee_{i < 0} \mathcal{F}_{i,-1}$, $E \in \mathcal{F}_{0,0}$;

- (e) $\lim_{n \rightarrow \infty} n^{-1} \log (dP_n/dQ_n)$ does not exist in $L^1(P)$.

(Condition (d) was overlooked in [1]. Also, all logarithms are to base 2.)

It is not hard to construct a double sequence $a_{n,j}$, $n, j = 1, 2, \dots$, such that

- (f) $a_{n,j} \geq 2$, $n, j = 1, 2, \dots$;
- (g) $|a_{n+1,j} - a_{n,j}| \leq [n \log (n+1)]^{-1}$, $n, j = 1, 2, \dots$;
- (h) $\lim_{n \rightarrow \infty} a_{n,j} = 2$ for each j ;
- (i) $\sum_{j=1}^{\infty} 2^{-j} a_{n,j} = 3$ for each n .

For each positive integer j , let P^j be the discrete probability measure on \mathcal{F} which assigns probability one to the sequence which is identically $2j$. From [1] we can construct a stationary discrete probability measure Q^j on \mathcal{F} with