*The Annals of Probability*
1977, Vol. 5, No. 2, 315–317

# NOTES

## CORRECTION TO "RADON-NIKODYM DERIVATIVES OF GAUSSIAN MEASURES"

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**Introduction.** J. R. Klauder kindly pointed out that the first statement of Theorem 11 of my paper [2] is incorrect. It was claimed incorrectly that if $h = h(t)$, $0 \leqq t \leqq T$ is a (strictly) increasing absolutely continuous function with $h(0) = 0$, then a necessary and sufficient condition that the Gauss–Markov process

$$X(t) = \frac{1}{(h'(t))^{\frac{1}{2}}} W(h(t)) , \qquad 0 \leqq t \leqq T \tag{1}$$

is equivalent to the Wiener process $W$, $X \sim W$, is that

$$\int_0^T \left[\frac{d}{dt}(1/(h'(t))^{\frac{1}{2}})\right]^2 dt < \infty . \tag{2}$$

The case

$$h(t) = t + t^{\frac{3}{2}} , \qquad 0 \leqq t \leqq T = 1 \tag{3}$$

gives an example where (2) fails although $X \sim W$. We will prove that the condition

$$\int_0^T h(t)\left[\frac{d}{dt}(1/(h'(t))^{\frac{1}{2}})\right]^2 dt < \infty \tag{4}$$

is necessary and sufficient for $X \sim W$. Note that (3) satisfies (4) but not (2). Theorem 1 of [2] gives a general condition for a Gaussian process to be equivalent to $W$ but the condition is difficult to apply in this case. Instead we use the elegant results of M. Hitsuda [1]. Note that [4] gives necessary and sufficient conditions among a restricted class of $h$ for $X \sim W$. Of course the exact scale normalization $1/(h'(t))^{\frac{1}{2}}$ in (1) is necessary for $X \sim W$ (e.g., note that $cW \sim W$ only for $c = 1$).

The error in the argument in [2] that $X \sim W$ implies (2) occurs in the ninth line from the bottom of page 344 where it is incorrectly claimed that $v' \in L^2[0, T]$ if $u'(\min(s, t))v'(\max(s, t)) \in L^2[0, T] \times [0, T]$.

The argument given for the converse assertion, that (2) implies $X \sim W$, tacitly assumes that $h$ is bounded and under this assumption is correct since then (2) implies (4) which implies that $X \sim W$. However for unbounded $h$, i.e., $h(T) = \infty$, e.g.,

$$h(t) = t/(1 - t) , \qquad 0 \leqq t \leqq T = 1 , \tag{5}$$