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THREE BOOKS ON STOCHASTIC INTEGRATION

K. L. CHUNG AND R. J. WILLIAMS, *Introduction to Stochastic Integration*. Birkhauser, Boston, Basel, Stuttgart, 1983, 191 pages, $19.95.

R. J. ELLIOTT, *Stochastic Calculus and Applications*. Springer-Verlag, New York, Heidelberg, Berlin, 1982, 302 pages, $48.30.

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REVIEW BY PHILIP PROTTER

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What do the terms stochastic integration and stochastic calculus connote? In the past they would refer to the Itô integral, a mathematically rigorous way to make sense of integration with respect to a Brownian motion. Times have changed. Now that the Itô integral has become a familiar if not ubiquitous object, "stochastic integration" has come to refer to an imposing panoply of abstract "French" probability theory, only now becoming accessible to the nonspecialist.

The stochastic integration of today, of course, has its roots in Brownian motion. The Wiener process, the mathematical model of Brownian motion, is indeed the wellspring of much of modern probability theory, perhaps due to its triple role of martingale, strong Markov process, and Gaussian process. It is the interplay of the martingale and Markov process properties that underlies the history of stochastic integration. By developing his integral in 1944 with stochastic processes as integrands, Itô [11] was able to study multidimensional diffusions with purely probabilistic techniques, an improvement over the analytic methods of Feller. Many diffusions can be represented as solutions of systems of stochastic differential equations of the form:

$$X_t = X_0 + \int_0^t \sigma(s, X_s)\, dB_s + \int_0^t b(s, X_s)\, ds,$$

where B is a Brownian motion (i.e., a standard Wiener process). This is still the primary method of studying multidimensional diffusions.

It was Doob, however, who stressed the martingale nature of the Itô integral [8]. Realizing that the martingale property of Brownian motion was the key in Itô's integral, Doob proposed a general martingale integral. To develop it, however, he needed to be able to decompose submartingales into the sum of a martingale and an increasing process. Meyer [16] found the right conditions under which this could be done and indicated how this might open the door to general stochastic integration ([16], page 204).

The door was not fully open, however. The rich structure of Brownian motion concealed a subtle distinction. Given a filtered probability space

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