# THE INFLUENCE OF MARK KAC ON PROBABILITY THEORY

By Harry Kesten

*Cornell University*

Kac's work covers a wide spectrum, from classical analysis through number theory, potential theory, pure probability theory, ergodic theory to statistical physics. One of the fascinating aspects of his work is that it demonstrates the interplay between these fields, and, despite the title of this paper, it would not do Kac's work justice to restrict ourselves here to its purely probabilistic aspects. Kac's enormous influence on statistical physics is discussed by Thompson in the companion article in this issue, and we shall therefore say essentially nothing about his work in that field. Nevertheless, it should be pointed out that, apart from the direct results obtained, Kac's activity in statistical physics also had the indirect influence on probability theory of convincing probabilists that there are exciting and important probability problems in statistical mechanics. Through his writings, lectures, and personal propaganda, Kac was instrumental in no small measure in the development of the very strong international group of probabilists who presently work on the borderline of statistical mechanics and probability theory.

An excellent survey of most of Kac's work was given in the introductory Commentary to the collection of selected reprints of Kac [1]. In addition Kac has written several illuminating autobiographical notes and a book ([2], [K165], [K182]).[1] I owe many of my remarks to these sources. In most cases I have not repeated all the references given in [1]; when possible I have added later references. Clearly all my remarks have been influenced by my tastes and limited knowledge. I apologize for misrepresentations and omissions of much work influenced by Kac.

For the purpose of this article it is convenient to divide Mark Kac's papers into the following (somewhat arbitrary) categories:

1. Probabilistic aspects of gap series and probabilistic number theory.
2. Interplay between probability theory and analysis.
3. Potential theory.
4. Limit theorems and invariance principles.
5. Feynman–Kac formula.

**1. Probabilistic aspects of gap series and probabilistic number theory.** Under this heading fall the limit theorems for lacunary series and number theoretic functions, which resemble the classical limit theorems (such as the three series criterion and the central limit theorem) both in form and in technique. Kac was brought to questions of this kind by his teacher Steinhaus

---