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TWO BOOKS ON RANDOM GRAPHS

BÉLA BOLLOBÁS, *Random Graphs*, Academic, London, 1985, 447 pages, $58.50 (paperback $29.95).

EDGAR PALMER, *Graphical Evolution: An Introduction to the Theory of Random Graphs*, Wiley, New York, 1985, 177 pages, $34.95.

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Problems concerning the structure of random media have challenged mathematicians and physicists for many years, and the last twenty years have witnessed much progress. Probabilists and statistical physicists have developed and refined techniques for approaching quantitative as well as qualitative questions, such as the nature of phase transition in physical systems and the bulk properties thereof. The limited extent of finite-dimensional space (usually $d = 2$ or $d = 3$) has constrained progress. In a parallel development described in these two books, combinatorial theorists have developed an intricate theory of certain random networks not subject to such constraints of dimensionality. Owing to the simplicity of definition of these so-called "random graphs," rich and complex discoveries have been made about their inner structures.

There are various different types of "random graphs," of which the following is perhaps the most basic. Take n vertices labelled $1, 2, 3, \dots, n$, and from the $\binom{n}{2}$ available unordered pairs draw N at random; join these pairs with edges to obtain a random graph ω_n. What are the properties of ω_n? In a paper which has since received much attention, Erdös and Rényi (1960) began to answer this question. They thought of such random graphs as growing organisms, observing the properties of ω_n as $n \to \infty$ when $N = N(n)$ is a prescribed function of n. Some examples of their findings are as follows:

(a) If $N(n) = o(n)$, then ω_n contains almost surely no cycle.

(b) If $N(n) = cn$ for some constant c, then the number ν_n of vertices in the largest component of ω_n is asymptotic (in appropriate senses) to $\alpha(c)\log n$ if $c < \frac{1}{2}$, $n^{2/3}$ if $c = \frac{1}{2}$ and $\beta(c)n$ if $c > \frac{1}{2}$, for constants α and β depending on c.

(c) If $N(n) = \frac{1}{2}n \log n + yn$, then the probability that ω_n is connected converges as $n \to \infty$ to $\exp(-e^{-2y})$.

Note that the expression "ω_n has property π almost surely" is used by random graph theorists in a nonstandard fashion to mean that $P(\omega_n$ has property $\pi) \to 1$ as $n \to \infty$.

In the above model the set of edges is a random set with prescribed cardinality. The following is an alternative model. Fix a number $p = p(n)$ in the interval $[0, 1]$. We examine each distinct pair of vertices from the vertex set $\{1, 2, \dots, n\}$ in turn, and we join this pair by an edge with probability p independently of all

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