

## KOLMOGOROV'S WORK ON ERGODIC THEORY

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Apparently the interests of Kolmogorov in ergodic theory had already started in the 1930s. In mathematical Moscow it was a period of construction of the foundations of the theory of stationary random processes. One might recall the paper by Khintchine [11] at that time dedicated to the spectral theory of such processes. Slightly later there appeared a very important paper by Kolmogorov [K116]<sup>1</sup> on regularity and extrapolation of stationary random sequences. The paper by Khintchine [10], where he gave a purely metric proof of the Birkhoff ergodic theorem, belonged to ergodic theory itself. In view of this paper the ergodic theorem on a.e. convergence of time averages is often called the Birkhoff–Khintchine theorem at least in the Russian literature. In the 1930s, the well-known paper by Krylov and Bogolyubov [12] on invariant measures for groups of homeomorphisms of topological spaces was written.

In the beginning of the 1930s, there appeared the famous paper by von Neumann [21], where the general notion of the metric isomorphism of one-parameter groups of measure-preserving transformations was introduced. Also in [21] von Neumann proved a basic theorem of metric isomorphism of ergodic dynamical systems with pure point spectrum. This theorem showed that for this class of systems the spectrum is the complete metric invariant. Since that time the problem of metric classification of dynamical systems became one of the central ones in ergodic theory.

The scientific activity of von Neumann was always under close attention. It is not surprising that this problem became well known quite soon in Moscow and several mathematicians spent a lot of effort trying to make some progress here. Unfortunately, I have no evidence whether the works of Morse, Hedlund and Hopf on ergodic properties of geodesic flows and on symbolic dynamics were widely discussed at that time.

For Kolmogorov the end of the 1930s was the beginning of his classical works on hydrodynamics and turbulence. His publication which can be considered as relating to ergodic theory goes back to 1937 (see [K84, K99]), where he exposed the Birkhoff–Khintchine theorem in probabilistic terms. At this time there were contacts between Kolmogorov and Rokhlin who at that time was a student of the mathematics department of Moscow University. For Rokhlin ergodic theory was the main domain of investigations at that time. As he recalled later Kolmogorov understood quite clearly the connections between the theory of stationary random processes and ergodic theory and, in particular, the

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<sup>1</sup>Reference citations preceded by K refer to the list of Kolmogorov's publications on pages 945–964.