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KOLMOGOROV AND THE THEORY OF MARKOV PROCESSES

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1. Beginning. In 1906–1907 Markov [59, 60] discovered that limit theorems for independent random variables can be extended to variables "connected in a chain." About the same time Einstein [31] started to study mathematically a physical phenomenon—the Brownian motion. The synthesis of both directions in Kolmogorov's celebrated paper [K28][1] was the beginning of the theory of Markov processes. (Kolmogorov called them stochastically determined processes. The name Markov process was suggested in 1934 by Khintchine.)

Today we distinguish Markov transition functions from measures on path space which can be constructed starting from such functions and which we call Markov processes. Measure theory on functional spaces did not exist in 1931, and Kolmogorov [K28] deals with Markov transition functions; paths are used only as a heuristic background to motivate definitions and assumptions.

2. Kolmogorov's program. In modern terms, Kolmogorov introduced a Markov transition function as a family of stochastic kernels $P(s, x; t, E)$ such that

$$\int P(s, x; t, dy)P(t, y; u, E) = P(s, x; u, E) \quad \text{for every } s < t < u. \tag{1}$$

Formula (1) is usually called the Chapman–Kolmogorov equation. Kolmogorov himself used the name of Smoluchowski who had written (1) in a special situation. After discussing various particular cases, Kolmogorov showed that the ergodic principle established by Markov for chains holds for broad classes of general transition functions.

The central idea of the paper is the introduction of local characteristics at each time t and the construction of transition functions by solving certain differential equations involving these characteristics. If the state space is at most countable, then the local characteristic at time t is given by a matrix $A_{jk}(t)$, and the corresponding differential equations have the form

$$\frac{\partial}{\partial t}P_{ii}(s, t) = \sum_j P_{ij}(s, t)A_{jk}(t) \tag{2}$$

and

$$\frac{\partial}{\partial s}P_{ik}(s, t) = -\sum_j A_{ij}(s)P_{jk}(s, t). \tag{3}$$

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[1]Reference citations preceded by K refer to the list of Kolmogorov's publications on pages 945–964.