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KOLMOGOROV'S EARLY WORK ON CONVERGENCE THEORY AND FOUNDATIONS

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1. Probability before Kolmogorov. When Kolmogorov was starting his mathematical career, nonmathematical probability was, as it still is, the study of various not very precisely defined real contexts. Some of these contexts gave rise to mathematical problems, in combinatorics for example, but it was not clear what an overall mathematical probability context would be or indeed whether one was possible. Poincaré had written in 1912 [13] "On ne peut donner une définition satisfaisante de la probabilité." It was typical of the writing of that time, and in fact of considerably more recent times, that the reader could not be certain whether the writer was thinking of probability as a nonmathematical or a mathematical subject in his statement. von Mises in 1919 [15] was clearer in what he deplored but was just as pessimistic, although more professorily ponderous: "In der Tat kann man den gegenwärtigen Zustand kaum anders als dahin kennzeichnen, dass die Wahrscheinlichkeitsrechnung heute ein mathematische Disciplin nicht ist." von Mises attempted to create his desired mathematical discipline but his theory of "collectives" was a confused although suggestive mixture of mathematical and nonmathematical contexts. In view of Kolmogorov's high opinion of von Mises a few explanatory remarks are appropriate here. Consider a sequence obtained by sampling a sequence of independent trials with a common distribution. (Note that sampling an infinite sequence is an unrealistic element of this analysis.) There are typical properties associated with such a sequence, as indicated for example by the law of large numbers. von Mises attempted to construct a basis for mathematical as well as nonmathematical probability by a formalization of these typical properties, by constructing an individual sequence with enough of these properties to be a model for a trial sequence. His original definition of such an individual sequence (a "collective") [15], was insightful of what seems to happen in sampling, but was either vacuous or meaningless when applied to an individual sequence, depending on the reader'sinterpretation of von Mises' words. His later definition [16] had too few properties to be useful. In any event such a construction, even if successful, would obviously be too awkward and too limited to be considered seriously as a useful basis for the extraordinary scope of modern probabilistic mathematical analysis. On the other hand, the formalization of such a sequence is an appealing conceptual problem which Kolmogorov discussed on several occasions, most recently in 1983 [K462].[1]

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[1]Reference citations preceded by K refer to the list of Kolmogorov's publications on pages 945–964.

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