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FRANK SPITZER'S WORK ON RANDOM WALK AND BROWNIAN MOTION

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Sums of independent identically distributed (i.i.d.) random variables were among the first subjects to be studied in probability theory. The sequence of partial sums $S_n := \sum_1^n X_i$, $n \geq 1$, for i.i.d. random variables is called a *random walk*. Around 1650, Fermat, Pascal and Huygens already solved a number of absorption problems for very special one- and two-dimensional random walks which arose in gambling and de Moivre obtained his local central limit theorem for sums of binomial random variables in 1733 [see Hald (1990) for the early history of probability and statistics]. Because random walks have been studied so long, our knowledge of their properties is very detailed. Nevertheless, random walks continue to be fascinating because elegant new properties are still being discovered.

Many early investigations dealt with limit theorems for S_n and, not surprisingly, these made strong assumptions on the common distribution of the X_i's. One of the directions of random walk theory has been to generalize limit laws such as the central limit theorem and the law of the iterated logarithm to settings with nonidentically distributed variables, or to finding higher order terms in the convergence to limit laws [e.g., the Berry–Esseen theorem, or expansions in the central limit theorem and various other topics which can be found in Petrov (1975)], or to prove refined invariance principles, which give information about the distribution of functionals of the whole sample path $\{S_k,\ k \leq n\}$, rather than about the distribution of S_n only [e.g., Donsker's theorem and Strassen's law of the iterated logarithm; see Billingsley (1968) and Bingham (1986), respectively]. Generally speaking, this type of result gives detailed and sometimes rather technical information about the random walk under rather strong assumptions on the distribution of F. It is often required that F have a second moment or regularly varying tails.

Frank Spitzer's interest was more in the direction of finding relationships which made *no a priori assumptions whatsoever* on the underlying distribution. This type of result relies solely on the fact that the X_i are independent and identically distributed. A classical example of this kind of approach is the determination by Lévy and Khinchine of all possible limit laws for $b_n^{-1}(S_n - a_n)$ for suitable constants a_n and b_n, and of necessary and sufficient conditions on the common distribution of the X_i for convergence of $b_n^{-1}(S_n - a_n)$ to any of the possible limit laws. [Gnedenko and Kolmogorov (1954) or Feller (1971, Chapter 17) are standard references for this general theory.]

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