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BOOK REVIEW

GREGORY F. LAWLER, *Intersections of Random Walks. Probability and Its Applications.* Birkhauser, Boston, 1991, 219 pages

REVIEW BY R. E. BRADLEY

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The contents of the book are as follows:

Chapter 1. Simple random walk
Chapter 2. Harmonic measure
Chapter 3. Intersection probabilities
Chapter 4. Four dimensions
Chapter 5. Two and three dimensions
Chapter 6. Self-avoiding walks
Chapter 7. Loop-erased walk

Random walks have fascinated and perplexed the mathematical community for about a century. Although there are a variety of complications and variations by means of which the basic model can be generalized, the behavior in the simplest case is already complex and surprising.

Consider a symmetric nearest-neighbor random walk on the integer lattice $\mathbf{Z}^d$. To what extent does the behavior of the walker depend upon the dimension d? On one hand, the mean-squared displacement is independent of dimension and $E(|S_n|^2) = n$ for every natural number n, where S_n is the walker's position after n steps. On the other, Polya proved in 1921 that if $d \leq 2$, such a walk is recurrent, whereas if $d \geq 3$, then the walk is transient.

The intersection properties considered by Gregory Lawler in *Intersections of Random Walks* are invariably dimension-dependent. The starting point for his investigations are the probabilities $p_n(x)$ that a walk beginning at the origin reaches the node $x \in \mathbf{Z}^d$ at the completion of its nth step. The first observation is that this probability can only be positive if the parity of n matches that of the sum of the components of x, in which case we write $n \leftrightarrow x$. The next observation is that the central limit theorem implies that $n^{-1/2}S_n$ converges in distribution to a normally distributed random variable in R^d.

A heuristic argument suggests that for large n, $p_n(x)$ should be approximately equal to

$$\overline{p}_n(x) = 2\left(\frac{d}{2\pi n}\right)^{1/2} \exp\left(\frac{-d|x|^2}{2n}\right).$$

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