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NOTE ABOUT "FIRST ORDER CORRECTION FOR THE HYDRODYNAMIC LIMIT OF SYMMETRIC SIMPLE EXCLUSION PROCESSES WITH SPEED CHANGE IN DIMENSION $d \geq 3$"

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We refer to [1] for notation.

In [1], we proved that the first order correction for the hydrodynamic equation of symmetric simple exclusion processes with speed change in dimension $d \geq 3$ is

$$\partial_t \rho^N = \Delta_u\big(\rho^N(1+\alpha\rho^N)\big) - \frac{\alpha}{N}\sum_{i,j=1}^{d} \partial_{u_i}^2\big(R_{ij}(\rho^N)\partial_{u_j}\rho^N\big),$$

where R_{ij} is a continuous function on (0, 1).

We now prove that $R = 0$, which means that there is no correction of order N^{-1} to the hydrodynamic limit.

Indeed, from Lemma 9.1 of [1], there exists a sequence of functions F_k^i that satisfies

$$\lim_{k\to\infty} E_{\nu_m}\left[W_{0,e_l}\sum_x \tau_x F_k^i\right] = -R_{i,l}(m)m(1-m),$$

where ν_m is the product Bernoulli measure of parameter m and W_{0,e_l} is the current between 0 and e_l. Notice that ν_m is translation invariant. Therefore, since our process is gradient, we obtain

$$E_{\nu_m}\left[W_{0,e_l}\sum_x \tau_x F_k^i\right] = E_{\nu_m}\left[\left(\sum_x \tau_x W_{0,e_l}\right)F_k^i\right] = 0,$$

and thus $R_{i,l}(m) = 0$.

THEOREM 1. *For symmetric simple exclusion processes with speed change, under diffusive rescaling, the density of particles $q^N(t,u)$ at time t around $u \in \mathbb{R}^d$ satisfies*

$$N\big(q^N(t,\cdot) - m(t,\cdot)\big) \to 0 \qquad \textit{for dimension } d \geq 3,$$

where $m(t,u)$ is the solution of the hydrodynamic limit $\partial_t m = \sum_i \partial_{u_i}^2(m(1+\alpha m))$.

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