

A CYCLICALLY CATALYTIC SUPER-BROWNIAN MOTION¹

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In generalization of the mutually catalytic super-Brownian motion in \mathbb{R} of Dawson and Perkins and Mytnik, a function-valued cyclically catalytic model X is constructed as a strong Markov solution to a martingale problem. Starting with a finite population X_0 , each pair of neighboring types will globally segregate in the long-term limit (noncoexistence of neighboring types). Also finer extinction–survival properties depending on X_0 are studied in the spirit of Mueller and Perkins. In fact, X_0 can be chosen in such a way that all types survive for all finite times. On the other hand, sufficient conditions on X_0 are stated for the following situation: given a type k and a positive time t , the k th subpopulation X^k dies by time t with a large probability, provided that its initial value X_0^k was sufficiently small.

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References

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