SIMULTANEOUS TREATMENT OF DISCRETE AND CONTINUOUS PROBABILITY BY USE OF STIELTJES INTEGRALS

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The object of this paper is to present several theorems pertaining to the probability that certain functions lie within certain intervals. The first theorem is a generalization of Markoff's Lemma, which is proven for the discrete and continuous cases by use of the accumulative frequency function and Stieltjes integrals. Tchebycheff's Theorem is obtained as a corollary to a very general theorem, the proof of which is based upon the first theorem. Other corollaries are given.

Three theorems, due to Guldberg, which follow are concerned with the probability that a non-negative chance variable be less than certain functions of the expected value of the variable. These are proved for the discrete and continuous cases by employing accumulative frequency functions and Stieltjes integrals. This is the first time, as far as the writer knows, the discrete and continuous cases for these theorems have been included in a single proof.

Theorem 1. If A denotes the expected value of the non-negative variable x and t is any number greater than 1, then the probability that $x \leq At^2$ is greater than $1-1/t^2$.

Proof: If x is a discrete variable with values at x_i, $(i = 1, 2, \cdots, n)$ with corresponding probabilities p_i, then it is understood that the probability that x takes other values is zero. If x is a continuous variable having a probability function defined over the interval (a, b), then it is understood that the probability that x lies outside of (a, b) is zero in case (a, b) is different from $(-\infty, +\infty)$. In both cases x is a continuous variable in the interval $(-\infty, +\infty)$. Let the probability that x lies in the interval $(-\infty, x)$ be $F(x)$, with $F(-\infty) = 0$ and $F(+\infty) = 1$ Then the probability that x lies in the interval (x_1, x_2) is

$$F(x_2)-F(x_1)+\frac{1}{2}\left\{F(x_2+0)-F(x_2-0)\right\}+\frac{1}{2}\left\{F(x_1+0)-F(x_1-0)\right\}$$