STIELTJES INTEGRALS IN MATHEMATICAL STATISTICS

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Introduction. Stieltjes integrals, introduced into analysis in 1894-5[1], play an increasingly important role not only in pure mathematics, but also in theoretical physics and in the theory of probability. In mathematical statistics, however, their use, it seems, still remains very limited. And yet, one of the most remarkable features of Stieltjes integrals is that they represent, as the case may be, an integral proper or a sum of an finite or an infinite number of *discrete* aggregates. Thus *the statistician is enabled to treat in a single formula a continuous, as well as a discontinuous distribution.* This means far more than a mere simplification of writing. In fact, since Stieltjes integrals have many properties in common with Riemann and Lebesgue definite integrals, we can use all known resources of the theory of definite integrals (mean-value theorem, various inequalities), and therefore readily obtain general results which, otherwise, require special (often complicated) proofs. The advantage of such a treatment is particularly evident in the theory of interpolation, approximation, and mechanical quadratures.

Hence, the object of this paper is to present a general exposition of the properties and applications of Stieltjes integrals. Many of the results stated below are well known[2], and the proofs may be omitted. Some results are believed to be new (for example, extension of Tchebycheff and Hölder inequalities) and may prove useful in mathematical statistics. We close, as an illustration, with the theory of interpolation, for here, even in recently published books, the continuous and discontinuous cases are treated *separately* while the underlying ideas are *identical.*

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1. Stieltjes: (a) Recherches sur les fractions continues, Oeuvres, v. II, p. 402-559; (b) Correspondence d'Hermite et de Stieltjes, v. II, p. 272, where these integrals are first mentioned in a letter (No. 351) to Hermite under date of October 25, 1892.

2. (a) Hobson, The Theory of Functions of a Real Variable, 2d. ed. (1921), v. I, p. 506-16, 605-09; (b) O. Perron, Die Lehre von den Kettenbrüchen (1913), p. 362-69.