BAYES' THEOREM [1]

By

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As for all established sciences, the typical problems of practical statistics have become inveterately attached to their several neat and convenient formulary solutions. To recall consideration of the basic reasoning underlying every-day statistical practice that applies to an elementary question may appear in the nature of an unnecessary disturbance of prevailing peace. If the experience of the writer is typical, however, vagueness or dubiousness of the premises inherent in a rule applied by rote will emerge to plague one in the conclusions, and a periodic return to fundamentals is as salutary for mental comfort as for the integrity of science itself. In what follows, an attempt will be made to go over the ground covered by Bayes' Theorem, and to point out its import for sound statistical reasoning. No claim is laid to mathematical originality at any specific points, but in the approach and synthesis will be found, we hope, a measure of instructive novelty.

A large class of statistical problems is typified in the following. A standard machine is known, from long experience, to produce a certain fraction P of imperfect products. What is the probability that in the next issue of n products, a fraction p will be imperfect?

We now present a related but not identical question. There is no available knowledge concerning the general practice of a machine; n products are examined and a fraction p found to be imperfect. What is the probability that the machine turns out generally a fraction P of imperfect products? The distinction between the two questions may be schematized as in Figure 1.

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