only an a priori determination, however uncertain, of the probability we are seeking. If we take the a priori probabilities ϖ for, and $(1 - \varpi)$ against, instead of μ and ν, so that

$$p = \frac{m + \varpi}{m + n + 1}, \qquad (137)$$

then we are certain to avoid the paradox of unanimity where it might do harm, without deviating so much as the mean error from the observation in the a posteriori determination.

Neither Bayes's rule nor this latter one can be of any great use; but we can always employ them, when the found probabilities can be looked upon as definitive results. On the other hand, the formula of the mean value *may* be used in all cases, if we interpret the paradox of unanimity correctly. Where the found probabilities are to be subjected to adjustment, the latter formula, as I have said, *must* be employed; nor can the other rules be of any help in the cases where observed probabilities have to be rejected on account of the skewness of the law of errors.

XVII. MATHEMATICAL EXPECTATION AND ITS MEAN ERROR.

§ 74. Whether the theory of probability is employed in games, in insurances, or elsewhere, in all cases nearly in which we can speak of a favourable event, the prediction of the practical result is won through a computation of the mathematical expectation. The gain which a favourable event entails, has a value, and the chance of winning it must as a rule be bought by a stake. The question is: How are we to compare the value of the latter with that of which the game gives us expectation? Imagine the game to be repeated, and the number of repetitions N to become indefinitely large, then it is clear, according to the definition of probability, that the sum of the prizes won, if each of them is V, must be pNV, when p indicates the probability. The gain to be expected from every single game is consequently pV, and this product of the probability and the value of the prize is what we call mathematical expectation.

The adjective "mathematical" warns us not to consider pV as the real value which the possible gain has for a single player. This value, certainly, depends, not only objectively on the quantity of good things which form the prize, but also on purely subjective circumstances, among others on how much the player previously possesses and requires of the same sort of good things. An attempt which has been made to determine by means of what is called the "moral expectation", whether a game is advantageous or not, must certainly be regarded as a failure. For it takes into account the probable change in the logarithm of