error" is $\sqrt{\frac{2}{\pi}}\lambda_2$. The only reason which may be advanced in defence of the use of this idea is that we are spared some little computations, viz. some squarings and the extraction of a square root, which, however, we rarely need work out with more than three significant figures.

IX. FREE FUNCTIONS.

§ 36. The foregoing propositions concerning the laws of errors of functions — especially of linear functions — form the basis of the theory of computation with observed values, a theory which in several important things differs from exact mathematics. The result, particularly, is not an exact quantity, but always a law of errors which can be represented by its mean value and its mean error, just like the single observation. Moreover, the computation must be founded on a correct apprehension of what observations we may consider mutually unbound, another thing which is quite foreign to exact mathematics. For it is only upon the supposition that the result $R = r_1 o_1 + \ldots r_n o_n = [ro]$ — observe the abbreviated notation — is a linear function of unbound observations only, $o_1 \ldots o_n$, that we have demonstrated the rules of computation (35)

$$\lambda_1(R) = r_1\lambda_1(o_1) + \ldots + r_n\lambda_1(o_n) = [r\lambda_1(o)] \tag{52}$$

$$\lambda_2(R) = r_1^2\lambda_2(o_1) + \ldots + r_n^2\lambda_2(o_n) = [r^2\lambda_2(o)]. \tag{53}$$

While the results of computations with observed quantities, taken singly, have laws of errors in the same way as the observations, they also resemble the observations in the circumstances that there can be bonds between them, and, unfortunately, there can be bonds between "results", even though they are derived from unbound observations. If only some observations have been employed in the computation of both $R' = [r'o]$ and $R'' = [r''o]$, these results will generally be bound to each other. This, however, does not prevent us from computing a law of errors, for instance for $aR' + bR''$. We can, at any rate, represent the function of the results directly as a function of the unbound observations, $o_1, \ldots o_n$,

$$aR' + bR'' = [(ar' + br'')o]. \tag{54}$$

This possibility is of some importance for the treatment of those cases in which the single observations are bound. They must be treated then just like results, and we must try to represent them as functions of the circumstances which they have in common, and which must be given instead of them as original observations. This may be difficult to do, but as a principle it must be possible, and functions of bound observations must therefore always have laws of errors as well as others; only, in general, it is not possible to compute these laws of errors correctly simply by means of the laws of errors of the