so that

$$s'_0 = 7,\ s'_1 = -8,\ \text{and}\ s'_2 = 70;$$

consequently

$$\mu_1 = 2650 - \tfrac{8}{7} = 2649.$$

$$\mu_2 = \tfrac{70}{7} - \left(-\tfrac{8}{7}\right)^2 = 9.$$

The mean deviation is consequently ± 3.

2. In an alternative experiment the result is either "yes", which counts 1, or "no", which counts 0. Out of $m + n$ repetitions the m have given "yes", the n "no". What then is the expression for the law of errors in half-invariants?

Answer: $\mu_1 = \dfrac{m}{m+n}$, $\mu_2 = \dfrac{mn}{(m+n)^2}$, $\mu_3 = \dfrac{mn(n-m)}{(m+n)^3}$, $\mu_4 = \dfrac{mn(m^2 - 4mn + n^2)}{(m+n)^4}$.

3. Determine the law of errors, in half-invariants, of a voting in which a voters have voted for a motion $(+1)$, c against (-1), while b have not voted (0), and examine what values for a, b, and c give the nearest approximation to the typical form.

$$\mu_1 = \frac{a-c}{a+b+c},\quad \mu_2 = \frac{ab + 4ca + bc}{(a+b+c)^2},\quad \mu_3 = \frac{(c-a)(ab + 8ca + bc - b^2)}{(a+b+c)^3},$$

$$\mu_4 = -\frac{((a+c)(a+b+c) - 4(a-c)^2)(a+b+c)(2a - b + 2c) + 6(a-c)^4}{(a+b+c)^4}.$$

Disregarding the case when the vote is unanimous, the double condition $\mu_3 = \mu_4 = 0$ is only satisfied when one sixth of the votes is for, another sixth against, while two thirds do not give their votes. If μ_3 is to be $= 0$, without a being $= c$, $b^2 - b(a+c) - 8ac$ must be $= 0$. But then $\mu_4 = -2\mu_2\left(\dfrac{a-c}{a+b+c}\right)^2$, which does not disappear unless two of the numbers a, b, and c, and consequently μ_2, are $= 0$.

4. Six repetitions give the quite symmetrical and almost typical law of errors, $\mu_1 = 0$, $\mu_2 = \frac{1}{3}$, $\mu_3 = \mu_4 = \mu_5 = 0$, but $\mu_6 = -\frac{4}{9}$. What are the observed values?

Answer: $-1, 0, 0, 0, 0, +1$.

VII. RELATIONS BETWEEN FUNCTIONAL LAWS OF ERRORS AND HALF-INVARIANTS.

§ 24. The multiplicity of forms of the laws of errors makes it impossible to write a Theory of Observations in a short manner. For though these forms are of very different value, none of them can be considered as absolutely superior to the others. The functional form which has been universally employed hitherto, and by the most prominent writers, has in my opinion proved insufficient. I shall here endeavour to replace it by the half-invariants.