

consideration which of them is the easiest to employ, and neither this nor the description of the forms of the laws of errors demands any higher qualification than an elementary knowledge of mathematics. But we must take into account also, how far the different forms are calculated to emphasise the important features of the laws of errors, i. e. those which may be transferred from the laws of actual errors to the laws of presumptive errors. On this single point, certainly, a more thorough knowledge of mathematics would be desirable than that which may be expected from the majority of those students who are obliged to occupy themselves with observations. As the definition of the law of presumptive errors presupposes the determination of limiting values to infinitely numerous approximations, some propositions from the differential calculus would, strictly speaking, be necessary.

III. TABULAR ARRANGEMENTS.

§ 9. In stating the results of all the several repetitions we give the law of errors in its simplest form. Identical results will of course be noted by stating the number of the observations which give them.

The table of errors, when arranged, will state all the various results and the frequency of each of them.

The table of errors is certainly improved, when we include in it the *relative frequencies* of the several results, that is, the ratio which each absolute frequency bears to the total number of repetitions. It must be the *relative* frequencies which, according to the law of large numbers, are, as the number of observations is increased, to approach the constant values of the law of presumptive errors. Long usage gives us a special word to denote this transition in our ideas: *probability* is the relative frequency in a law of presumptive errors, the proportion of the number of coincident results to the total number, on the supposition of infinitely numerous repetitions. There can be no objection to considering the *relative frequency* of the law of actual errors as an approximation to the corresponding *probability* of the law of presumptive errors, and the doubt whether the *relative frequency* itself is the best approximation that can be got from the results of the given repetitions, is rather of theoretical than practical interest. Compare § 73.

It makes some difference in several other respects — as well as in the one just mentioned — if the phenomenon is such that the results of the repetitions show qualitative differences or only differences of magnitude.

§ 10. In the former case, in which no transition occurs, but where there are such abrupt differences that none of the results are more closely connected with one another than with the rest, the tabular form will be the only possible one, in which the law of errors can