

# THE GENERALIZATION OF STUDENT'S RATIO\*

By

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The accuracy of an estimate of a normally distributed quantity is judged by reference to its variance, or rather, to an estimate of the variance based on the available sample. In 1908 "Student" examined the ratio of the mean to the standard deviation of a sample.<sup>1</sup> The distribution at which he arrived was obtained in a more rigorous manner in 1925 by R. A. Fisher,<sup>2</sup> who at the same time showed how to extend the application of the distribution beyond the problem of the significance of means, which had been its original object, and applied it to examine regression coefficients and other quantities obtained by least squares, testing not only the deviation of a statistic from a hypothetical value but also the difference between two statistics.

Let  $\xi$  be any linear function of normally and independently distributed observations of equal variance, and let  $s$  be the estimate of the standard error of  $\xi$  derived by the method of maximum likelihood. If we let  $t$  be the ratio to  $s$  of the deviation of  $\xi$  from its mathematical expectation, Fisher's result is that the probability that  $t$  lies between  $t_1$  and  $t_2$  is

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<sup>1</sup>*Biometrika*, vol. 6 (1908), p. 1.

<sup>2</sup>*Applications of Student's Distribution*, *Metron*, vol. 5 (1925). p. 90.