ON SYMMETRIC FUNCTIONS OF MORE THAN ONE VARIABLE AND OF FREQUENCY FUNCTIONS

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In a paper published in this journal[1] the writer has developed a simple differential operator method for expressing any symmetric function of the n variates $x_1, x_2, \cdots\cdots, x_n$ as a rational, integral, algebraic function of the power sums $s_1, s_2, \cdots, s_w$ where w is the weight of the symmetric function and

$$s_k = \sum x_i^k = x_1^k + x_2^k + \cdots\cdots + x_n^k.$$

The transformation to moments is then simply a matter of recognizing that $n\mu_k' = s_k$ if μ_k' is the kth moment of the n variates with respect to the origin from which they are measured. If the origin is at the arithmetic mean of the n variates the prime may be dropped and then $n\mu_k = s_k$.

In the above mentioned paper the variates x_i are of the serial distribution type, but, of course, not necessarily integers. The extensions to the case of more than one set of variates and to frequency functions now suggest themselves. It is the purpose of this note to discuss these problems simultaneously.

Suppose that two sets, of n variates each, $x_1, x_2, \cdots, x_n$ and $y_1, y_2, \cdots, y_n$ are given and that x_i, y_i $(i = 1, 2, \ldots\ldots, n)$ are corresponding pairs. Modifying the partition notation used in the previous paper the symmetric function to be considered may be written in the form $(a_1^{n_1} a_2^{n_2} a_3^{n_3} \cdots\cdots \cdot b_1^{m_1} b_2^{m_2} b_3^{m_3} \cdots)$ i.e. the

sum of all such terms as

$$x_1^{a_1} x_2^{a_1} \cdots x_{n_1}^{a_1} x_{n_1+1}^{a_2} \cdots x_{n_1+n_2}^{a_2} \cdots\cdots y_1^{b_1} y_2^{b_1}$$
$$\cdots y_{m_1}^{b_1} y_{m_1+1}^{b_2} \cdots y_{m_1+m_2}^{b_2} \cdots$$

[1] Symmetric Functions and Symmetric Functions of Symmetric Functions, Vol. II. No. 2 (May, 1931), pp. 102-149.