THE LIMITS OF A MEASURE OF SKEWNESS

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The measure of skewness

$$s = \frac{\text{mean} - \text{median}}{\text{standard deviation}}$$

is sometimes recommended because of its simplicity. Obviously neither this nor any other statistic can be of much value until something at least is known of its distribution in samples from populations of some plausible form. For populations near the normal form the inefficiency of the median as a statistic of location suggests that the standard error of s may be considerably greater than that of μ_3/σ^3. We know of no investigation of the sampling distribution of s. Apparently even the range is unknown. The object of the present note is to show that s necessarily lies between -1 and 1.

The proof consists of three successive transformations of the sample, each increasing s, which nevertheless in the end remains less than unity.

1. Without loss of generality let us suppose that the median is zero and that the mean $\bar{x}$ is positive. Taking

$$\sigma^2 = \frac{\Sigma(x-\bar{X})^2}{n} = \frac{\Sigma x^2}{n} - \bar{x}^2,$$

n being the number of observations, which we suppose odd, we have

$$s = \bar{x}/\sigma.$$

If a negative observation $-a$ be replaced by zero, the mean is increased by a/n. In the second of the expressions above for σ^2, the mean of the squares is diminished by a^2/n, while on account of the change in the mean, a further subtraction is made. Thus σ diminishes. Hence s increases if we alter the distribu-