A STUDY OF THE DISTRIBUTION OF MEANS ESTIMATED FROM SMALL SAMPLES BY THE METHOD OF MAXIMUM LIKELIHOOD FOR PEARSON'S TYPE II CURVE

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The object of this paper is to study the distribution of estimates of the parameter of location for Pearson's Type II Curve, estimated by the method of maximum likelihood from small samples.

R. A. Fisher has assumed,[1] and Professor Hotelling has proved[2] that in large categories of cases the distribution of an optimum statistic approaches normality as the sample size increases. This normality has been assumed to hold for optimum statistics in general whether calculated from large samples or small ones, and it has also been assumed that optimum statistics have minimum variance and always give better fits than do statistics calculated by the method of moments. That this is the case whenever the sample is large and the distribution of optimum statistics normal is made plausible by the reasoning of R. A. Fisher.[3] In case the sample is small, however, there may be reason to doubt that the normality of distribution of optimum statistics holds, and that the other conclusions hold. It is with this phase of the subject that we shall be concerned in what follows.

Before entering into the topic under discussion it will be convenient to review some of the more elementary facts regarding the curve with which we are to be concerned. We shall take first the general equation for the curve in the form,

$$(1) \qquad y = y_0\left(1 - \frac{(x-m)^2}{a^2}\right)^p$$

[1] On the Mathematical Foundations of Theoretical Statistics, R. A. Fisher, Phil. Trans. Series A, Vol. 222, 1922. Pp. 309-368.

[2] The Consistency and Ultimate Distribution of Optimum Statistics. Harold Hotelling, Trans. Amer. Math. Soc. Vol. 32, No. 4. Pp. 847-859.

[3] Ibid, Pp. 328-368.