CONCERNING THE LIMITS OF A MEASURE OF SKEWNESS

By RAYMOND GARVER
University of California at Los Angeles

In a recent note in the Annals of Mathematical Statistics,* Hotelling and Solomons devised an ingenious method of showing that the measure of skewness s defined by the equation

$$s = \frac{\text{mean - median}}{\text{standard deviation}}$$

cannot be greater than unity in absolute value. I am venturing to offer another proof of the same fact, which seems to me to be of interest because it employs an important and well-known algebraic inequality.

With Hotelling and Solomons, I shall assume that we are concerned with n readings, or x's, with median zero and mean $\bar{x}$, where $\bar{x}$ of course is $\Sigma x/n$ We may show that the absolute value of s cannot be greater than one by showing that $1/s^2$ is not less than one. Making obvious substitutions, we must then show that

$$\frac{n\Sigma x^2}{(\Sigma x)^2} \geqq 2.$$

Now according to a known theorem if $a, b, \cdots, k$ are n positive numbers, and if m is a number not lying between zero and one, then

$$\frac{a^m + b^m + \cdots + k^m}{n} \geqq \left(\frac{a + b + \cdots + k}{n}\right)^m.$$

*Vol. 3, no. 2, May, 1932, 141-2.