

## ON THE CORRELATION BETWEEN CERTAIN AVERAGES FROM SMALL SAMPLES\*

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1. *Introduction.* It is well known that no correlation exists between the arithmetic mean and standard deviation of samples drawn at random from a normal universe. However, there seems to be in the literature no treatment of the correlation between other averages either for normal or non-normal universes. In the present paper, a few simple theorems are established which make possible the determination of the type of regression of the median on the arithmetic mean, of the range on the median, and of the range on the arithmetic mean. In case the regression is linear, the coefficient of correlation may be computed.

We shall understand a probability function  $f(x)$  of a real variable  $x$  to be, for all values of  $x$  on a range of  $\mathcal{R}$  a single-

valued, non-negative, continuous function with  $\int_{\mathcal{R}} f(x) dx = 1$ .

Then  $\int_a^b f(x) dx$  is the probability that a value of  $x$  chosen

at random lies in the interval  $(a, b)$  where  $a$  and  $b$  are in  $\mathcal{R}$  and  $a < b$ ; and  $f(x) dx$  is, to within infinitesimals of higher order, the probability that a value of  $x$  chosen at random lies in the interval  $(x, x+dx)$ . It will prove convenient to classify probability functions according as  $\mathcal{R}$  is the range  $(-\infty, \infty)$ ,  $(0, \infty)$ , or  $(0, k)$ ,  $k > 0$ . In accord with this classification,<sup>1</sup> we shall refer to probability functions as of the first, second, and third kinds respectively. In a similar manner, we define a probability function  $F^2(x, y)$  of two independent variables.

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<sup>1</sup>Cf. L. Bachelier, *Calcul des Probabilités*, p. 155.